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Optimal design of intelligent sensors in improving the safety of steel structures

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Abstract:

Instrument health monitoring can identify the behavioral characteristics and detect the possible failure of structures during their lifetime. Structural health monitoring is a process in which various issues are addressed, including the diagnosis and location of damage to structures. A structural health monitoring system receives information about the instrument's behavior from the sensor network. Because installing sensors, receiving information from them, recording and processing data, and concluding is an essential process in monitoring, lack of data leads to incorrect or insufficient results, and additional data confuses the analysis. And data processing. Also, increasing the number of sensors increases the cost of monitoring the structure. Therefore, the optimal design of the sensors network to correctly use sensors in terms of type, number, and critical location is essential.

Introduction:

This research extracts a method to optimally design a network of sensors for a health monitoring system. The optimal positioning of sensors is done according to their ability to detect modal parameters. It was shown that the optimal sensor placement method depended on the sensor layout irrespective of the same quantity of sensors.[1] This research determines sensors' optimal number and location using different optimization algorithms and the proposed modified discrete differential evolution algorithm. Then, using finite element modeling and structural analysis under impact load, the failure characteristics are extracted from the optimal location of the sensors. With the help of the neural network hierarchy, the possible failures of the members are estimated. Comparing the proposed method of discrete differential evolution algorithm and existing methods, such as sequential amplification of the sensor and advanced genetic algorithm for optimal sensor design, show that the proposed way is more efficient and suitable than existing methods. Also, in this study, using the neural network hierarchy and innovating the donor index as the input of the neural network, the damage detection technique was very effective in detecting failures in structures. The results show that combining the reduced frequency response and the acceleration response based on the reduced time is much better than either of the indicators alone [2].

Research history

Index based on the natural frequency

Theoretically, when a structure is damaged, the natural frequency decreases. This hypothesis has led to the widespread use of natural frequencies to detect malfunctions in research and accurate malfunction. The reason for the popularity of natural frequency is easy extraction because sometimes, even with a highly reliable sensor, it can illustrate the failure in structures. However, the natural frequency is not enough to feature the failure in facilities. It can not give complete information about the changes in different structure parts. As a result, it is difficult to determine the location and severity of failure using this indicator. Another disadvantage of natural frequency is the lack of correct response in symmetric failures in symmetric structures. Also, the natural frequency is strongly influenced by environmental conditions such as changes in humidity and temperature. These factors and other disadvantages listed in the reference [3-5] make using natural frequency alone to diagnose failure unsatisfactory.

Some researchers have conducted satisfactory studies on the natural frequency despite the above and determined the location and size of cracks in the beam using natural frequencies. Their research established a logical relationship between failure intensity and natural frequency reduction. However, natural frequency change was not related to failure locations. Garski et al. [10] and Kim et al. researched natural frequency.

Damping-based index

Damping is another modal parameter investigated as a failure feature in a few studies—the damping ratio increases when an instrument is damaged [11]. The biggest drawback of the damping-based fault detection method is the uncertainty of achieving damping in actual structures. For this reason, damping-based damage detection has not expanded significantly. Despite limited research, studies have often failed to find a logical link between damage and attenuation change. With a dynamic experiment on a steel bridge, saline and Baldwin sought to change the damping as a failure indicator [12].

Nevertheless, these researchers could not obtain satisfactory results. "Abdolrazaq and Choi" experimented with reinforced concrete beams. They found that the second and third modes' damping increases significantly with the increasing severity of the damage.

Index based on mode shape

Mod shape is a unique feature of any structural system. Much research has been done to identify damage based on the shape of the mode, whether directly measured or derivatives of the model's shape. These studies have suggested that the model's shape is a more reliable failure indicator than natural frequency and damping. Given that the model's shape expresses the pattern of relative displacement of all parts of a structure in that model's shape, it can provide comprehensive information. If part of the structure is damaged, the model's shape in the vicinity of the damage will change. As a result, by comparing two series of mode shape data before failure and after it (direct or its derivatives, failure can be identified. Two standard methods for comparing the shape of modes are MAC and COMAC. These two criteria for determining damage Have been used in many numerical examples. [14] Examples of direct use of the mod shape in diagnosing damage can be found in the references. The curvature of the model's shape is more sensitive to damage than the shape of the model itself, so the failure index is more appropriate. Using the curvature of the shape of the mode to locate the damage has shown satisfactory results. [18, 19] Modal shape curvature has been used to identify the damage. Another group of studies has used a dynamic softness matrix, inversely defined as a structural stiffness matrix, to determine damage. Satisfactory, although the majority of these studies are numerical and only one swimming failure scenario Sai is located. Reference [20] introduced the Comprehensive Softness Index, which was based on the modal softness matrix of the structure. Contact [21] used a new failure index, modal index, and softness methods to identify damage. In modal strain detection methods based on modal strain energy, modal strain energy changes before and after failure are examined. Some studies show that the modal strain energy index gives better answers to detect failure among all the indicators of failure based on the model's shape. However, this method still has drawbacks, such as reporting faulty faults, failing to report low-intensity defects and sensitivity to disturbances. The authorities used modal strain energy to detect damage to structures.

Research method:

In this part of the article, determining the optimal number and location of sensors is vital for an efficient health monitoring system. In this research, the problem of the optimal location of sensors (OSP) is defined as follows:

They are placing "**m"** sensors in the structure so that the software results best match the shape of the demand modes.

The mathematical model of OSP is defined based on the shape of the modes as follows:

$$\ddot{q} + \ q + perscritM^{-1}Kq = F \quad (1)$$

Where $q \ \in \ R^N$Is the mode coordinate vector K, M, and $C \ \in \ R^{m\times N}$Are the mass, $y = \Phi\text{q} +$ stiffness, and damping of the structured cap$\in \quad ^{m\times N}$, respectively, of the shape of the structural modes of the column. It is normalized by mass. $F \epsilon R^M$ It is the force vector. The sign -1 indicates the inverse, and the T indicates the transposition. $Y \ \in R^M$Is the normal coordinate vector and $\varepsilon \ \in R^M$. The equation is the sensor perturbation vector, usually a Gaussian function with zero mean and positive covariance. As a result, the OSP problem can be defined as follows:

Select "**m"** rows from the shape matrix of mode Φ, which best fits with a set of shapes of demand modes "**n."**

In this section, to optimize the sensors, the model of a one-story frame with a span modeled by Abacus software was examined as the basic structure. The first part of the research tests the basic model's various optimal sensor location methods. In addition to conventional methods, the developed genetic algorithm has been used for optimal sensor location. In the second part of the research and the continuation of the studies, due to the increase in the number of openings and floors of the CFS frame, Appendix software replaced Abacus to increase the speed of operation and modeling and analysis efficiency. Then, selecting two appropriate objective functions from the first part, the proposed improved discrete differential evolution algorithm was used to optimize the sensors. Both sections are discussed in detail in later sections.

Optimal sensor design in one-story frame and one aperture

Study model

The frame used in this research is a one-story frame with a span that is modeled in Abacus software. The brace is also made of a cross strap. The shell element was used for modeling for all members. This comparison shows a good adaptation of the software model to the laboratory model. After model validation, modal analysis was performed on the frame in Abacus software to complete the optimization process.

Six forms of the initial model of the frame were extracted. Four forms of the first mode are related to the buckling of the braces and do not affect the overall analysis of the modal of the frame. As a result, this form was abandoned in the optimization process. Thus, the first form of lateral and vertical frames was selected as the demand modes, shown in Figure (1). After extracting the shape of the demand modes, two types of initial compositions were considered for placing the sensors on the frame, which are shown in Figures (2) and (3), respectively:

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1- Sensors can be installed only in external cords and up and down trucks.

2- Sensors can be installed in all horizontal and vertical elements.

In the first type, 84 nodes were considered the initial location of the sensors. Considering each node's two horizontal and vertical transmission freedom directions, there are 168 degrees of initial freedom to place the sensor. In the second case, knots increased from 174 degrees of freedom to 348. Now that we have the form of demand modes and the initial location of the sensors, it is time to use the optimization algorithm to determine the optimal location of the sensor, which is done in the next section.

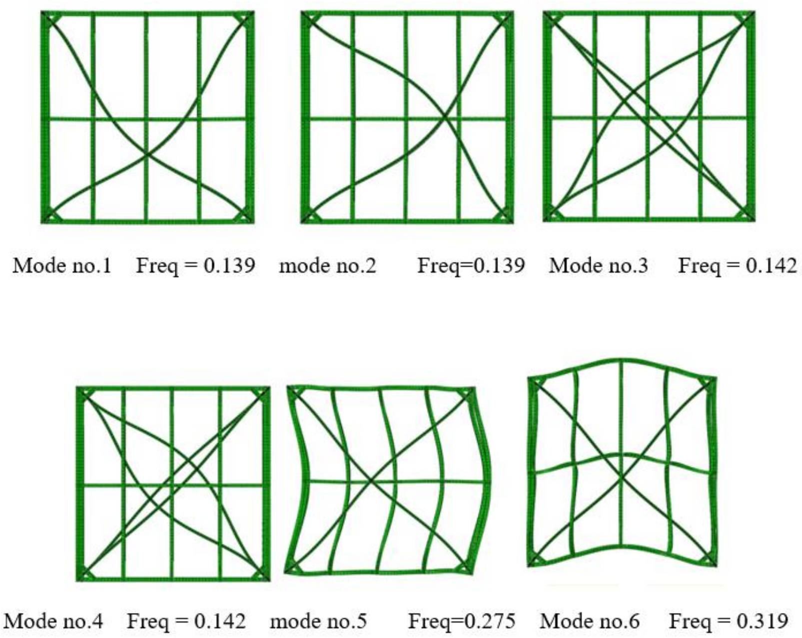

Figure 1: Shape of frame modes

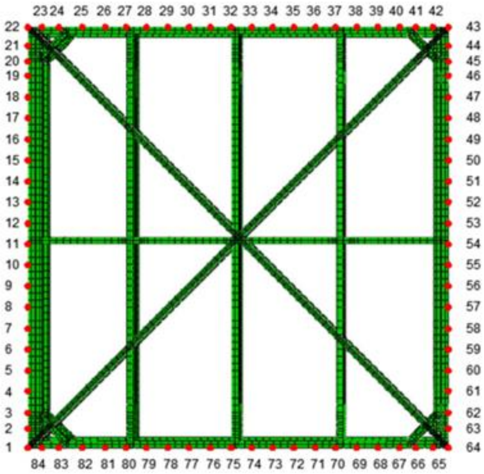

Figure 2: The first combination to locate the sensors

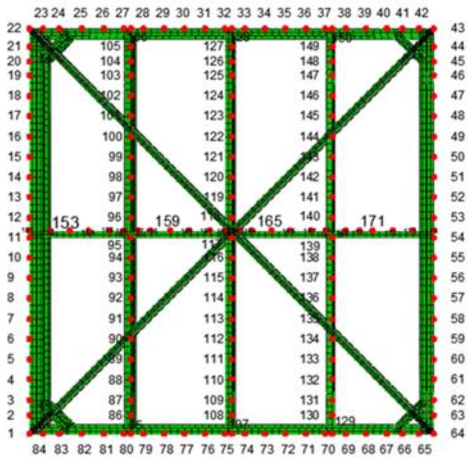

Figure 3: The second combination for locating the sensors

Implement standard methods used for OSP

This section briefly describes the standard methods used to determine the optimal location of the sensors. This study aims to evaluate the efficiency of conventional methods in determining the optimal location of sensors in a light steel frame. The selection of a suitable objective function for more extensive research in the second part is also made.

1- EFI: First, several modes of the structural analytical model are selected as the shape of the demand modes. After that, many degrees of freedom is chosen as the initial place to place the sensors in the structure. This number is selected large enough to cover all forms of demand modes. The number of degrees of freedom in which the sensors are placed must be greater than the number of selected odes. Thus we will have the following:

$$y = \Phi q + \varepsilon \ (2)$$

Where $y \epsilon R^{m\times 1} the$ vector contains sensor outputs in all locations nominated for sensors, and m is the number of selected locations. $\Phi \in R^{m\times n}$ Is the shape matrix of demand modes in candidate locations, in which n represents the number of shapes of demand modes. $q \in R^{n\times 1}$ is the coordinate vector of the demand modal parameter. € also expresses perturbation, modeled as Gaussian perturbation with zero medians. The above relation means that in a linear structure, the structure's response in the time domain is obtained by the linear combination of the shape of the modes multiplied by a coefficient vector such as q. The candidate locations of the sensors are selected so that the matrix columns have linear independence. The linear independence of the shape of the modes indicates that the sensors' outputs can estimate the shape of the modes at any point in time. Theoretically, the minimum number of required sensors equals the number of shapes of demand modes. However, due to the uncertainties of the instruments, this number should be considered more. According to the above, if out of s, the candidate's location for the sensors is only in the sensor's location, the purpose of the location is to determine this location. Considering relation (2), we will have the following:

$$c = E\left[(q-\hat{q})(q-\hat{q})^T\right] = \left[\frac{1}{\sigma^2}\,\phi^T\,\Phi\right]^{-1} = F^{-1} \qquad (3)$$

$$q_i = \left[\Phi_i^T \Phi_i\right]^{-1} \Phi_i^T\, y_i \qquad 1 \le i \le \binom{m}{k}$$

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Where c is the error covariance matrix E and must be minimized, F is the Fisher information matrix. $\hat{q}$The vector is estimated and is. According to the above relation, minimizing the estimation error q is equivalent to maximizing the FIM matrix or, in other words, the determinants of the FIM matrix. As a result, the optimization problem changes as follows: Find the location of the sensor m from among the candidate locations so that the determinants of the FIM matrix are maximized. After presenting this objective function, Kamer removed one of the candidate locations from the process with the least involvement in maximizing the FIM matrix determinants. The selection of the least effective sensor in maximizing the FIM matrix determinants is based on the EID linear independence distribution vector. To optimize the determinants of the FIM matrix, the sensor with the lowest EID must be removed at each step.

$$(4)E_D \ = diag\ (\Phi\ [\Phi^T \Phi]\ \ \Phi^T\ ) = diag\ (QQ^T)$$

$$P = \Phi\ [\Phi_m^T\ \Phi_m]^{-1} \Phi^T$$

It is a converted matrix Φ that decomposes as Φ = QR. Where Q is an er-normal matrix and R is an upper triangular matrix. The number of remaining locations calculations is repeated, and the sensor is removed with minimal participation. This process continues until the initial number of locations is reduced to the desired optimal number of m. In this method, the distribution of sensors will be relatively uniform. The results of this method for the sensors of the first and second compounds are shown in Figures (4) and (5). The sensors are concentrated in the most modest places. In the first type of layout, the distribution is relatively uniform. However, in the second type of layout, the sensors cover the shape of the first mode more than the shape of the second mode.

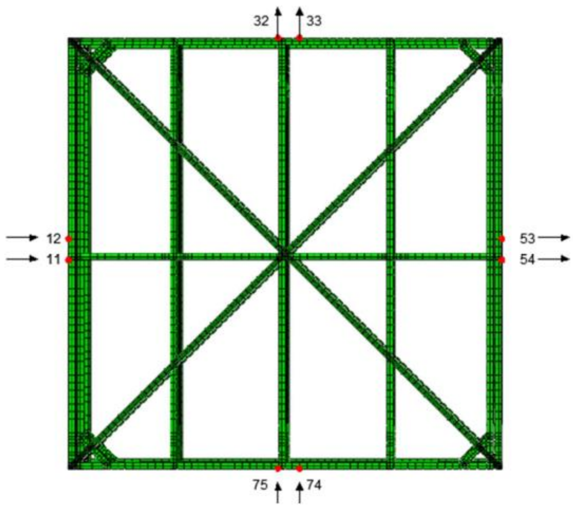

Figure 4 - Optimal location of sensors based on the EFI method in the first combination for eight sensors

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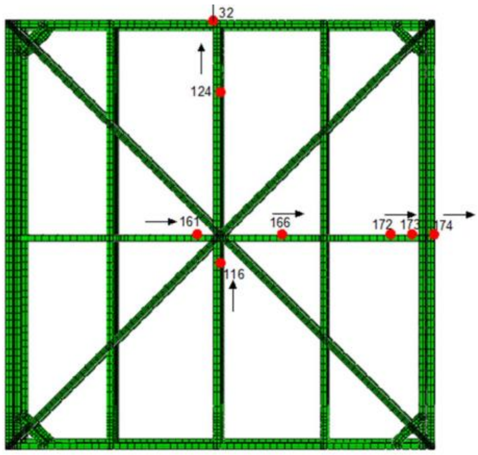

Figure 5 - Optimal location of sensors based on the EFI method in the second combination for eight sensors

Definition of variables:

Optimizing variables are possible degrees of freedom for the sensors in the optimal location of sensors. In the optimization problem, the m sensor is installed in the frame. The number m = 2,4,8 is selected to investigate the effect of the number of sensors on the results. In the advanced genetic algorithm method, each chromosome consists of m genes. A set of chromosomes in a matrix forms the initial population. This type of coding takes up less space. It is more suitable for optimizing sensors than the previous coding based on zeros and ones. An example of this coding, called decimal, form = 8, can be seen in Table (2). The first improvement to the typical genetic algorithm has been applied here. The coding has been improved from zero and one to decimal. For example, from Table 2, it can be seen that eight sensors are considered, whose initial location on the first chromosome is in degrees of freedom 11, 53, 12, 54, 116, 117, 157, and 159.

Table 1- Optimal location of sensors for the second combination

Number of sensors			**Method**	**Number**
2	**4**	**8**		
32 172 y x	116 166 172 32 y y x x	124 161 166 172 173 174 32 116 y y y x x x x x	EFI	**1**
32 172 y x	116 166 172 32 y y x x	116 125 161 166 167 172 174 32 y y y x x x x x	EFI - DPR	**2**
101 102 x x	102 145 146 101 x x x x	100 101 102 133 134 145 146 94 x x x x x x x x	EVP	**3**
88 131 x x	102 103 131 88 x x x x	89 102 103 131 133 146 147 88	NODP	**4**
12 138 x y	76 82 138 12	76 82 126 128 138 161 174 12	Min MAC	**5**

	x x x y	X x x y y y x x		
13 37 x y	37 60 128 13 x y y y	37 60 126 127 128 161 174 13 x y y y y y x x	E min MAC	**6**
12 96 x x	96 154 155 12 x x x x	152 154 155 157 158 159 12 96 x x x x x x x x	Vr	**7**
96 166 x x	140 166 173 96 x x x x	95 96 140 166 167 172 173 53 x x x x x x x x	MSSP	**8**
166 172 x x	167 172 173 166 x x x x	161 166 167 172 173 174 151 152 x x x x x x x x	DPR	**9**

Table 2 - Decoding coding for advanced genetic algorithm for m = 8

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**Number of genes**	**population**
11	12	53	54	116	117	158	159	Chromosome 1	
13	15	61	72	100	121	154	160	Chromosome2	
...	...	...	...	...	...	...	...	...	
10	13	54	63	109	131	159	166	Chromosome	

Analysis of optimization results with objective function based on MAC

Figures (6) to (11) show the maximum change curve of the MAC matrix's non-diameter diameter for adding a sensor of 2 to 25 to the CFS frames for the three different methods FSSP, MDDE, and IGA. A complete search was performed to extract the graphs, considering all the degrees of freedom the sensor could be placed. The graph's vertical axis represents the objective function's value, and the horizontal axis represents the number of sensors.

The optimal number of sensors based on the proposed algorithm for the MAC objective function is shown in Table (3) for each frame. The optimal number of sensors is selected based on the diagram of changes in the objective function relative to the number of sensors. The value of the diagram, then the number of sensors, has a constant trend so that the addition of a sensor in exchange for no change can be made. Attention to the value of the objective function is not economically justified. The degrees of optimal freedom are the design variables or locations of the sensors that have been optimized during the optimization process.

Table 3: Number of optimal sensors and values of Objective 1 function obtained from sensor optimization

Percentage of improvement		**The value of the objective function 1**			**The optimal number of sensors**	**CFS frame model**
IGA algorithm ratio	Compared to the FSSP algorithm	IGA algorithm	FSSP algorithm	MDDE algorithm		
78	84.3	0.05	0.07	0.011	8	**1S1S**
45.8	76.4	0.048	0.11	0026	12	**2S1S**

37.3	66.2	0.083	0.1539	0.0521	15	**3S1S**
40.7	46.7	0.013	0.015	0.008	9	**1S2S**
18.7	41.1	0.041	0.057	0.034	15	**2S2S**
17.7	25.9	0.077	0.086	0.064	18	**3S2S**

In all frames, it can be seen that for the various numbers of sensors, the values of the target functions obtained from the proposed MDDE method are smaller than those of the target functions obtained from the other two methods. As a result, it can be seen that the proposed method is more efficient than the conventional FFSP method and the well-known IGA method. Therefore, it can be an excellent alternative to past methods. The highest percentage of improvement is related to the ISIS frame, and the lowest gain is associated with the 3S2S frame. According to the graphs and tables, it can be seen that although FFSP works better in all frames, it is still less efficient. The genetic algorithm is more efficient than the MDDE method in almost all cases.

According to the graphs in all three methods, by increasing the number of sensors, the value of the objective function approaches zero, which is the optimal value. However, from a number onwards in all three methods, the reduction in the value of the objective function is so slight that it would not be economical to increase the number of sensors to provide this reduction. This number is less for the MDDE method than IGA and FSSP methods. The smaller number indicates that in the MDDE method, a more appropriate amount of the objective function can be achieved at a lower cost.

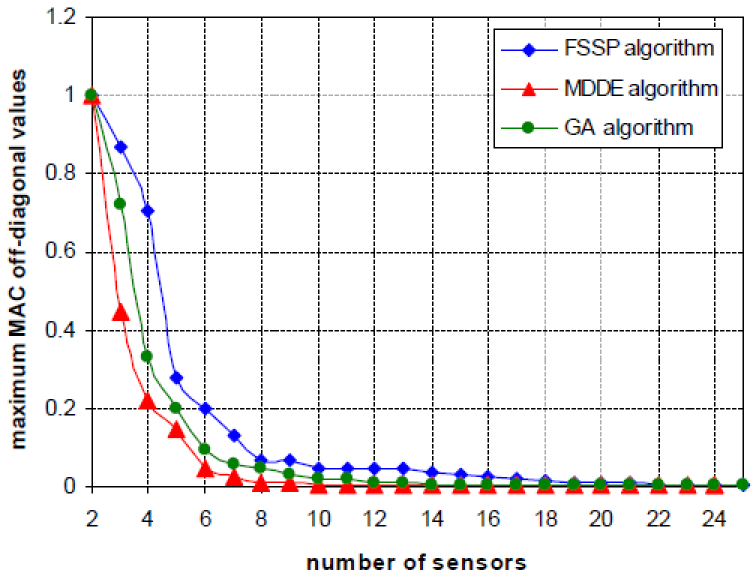

Figure 6: Diagram of changes in the MAC objective function based on the number of sensors for the 1S1S frame

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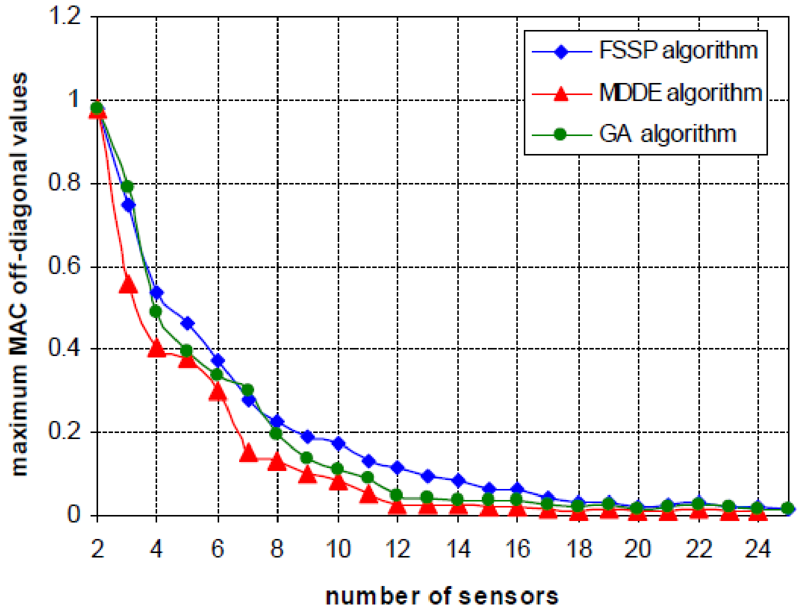

Figure 7: Graph of changes in the MAC objective function based on the number of sensors for the 2S1S frame

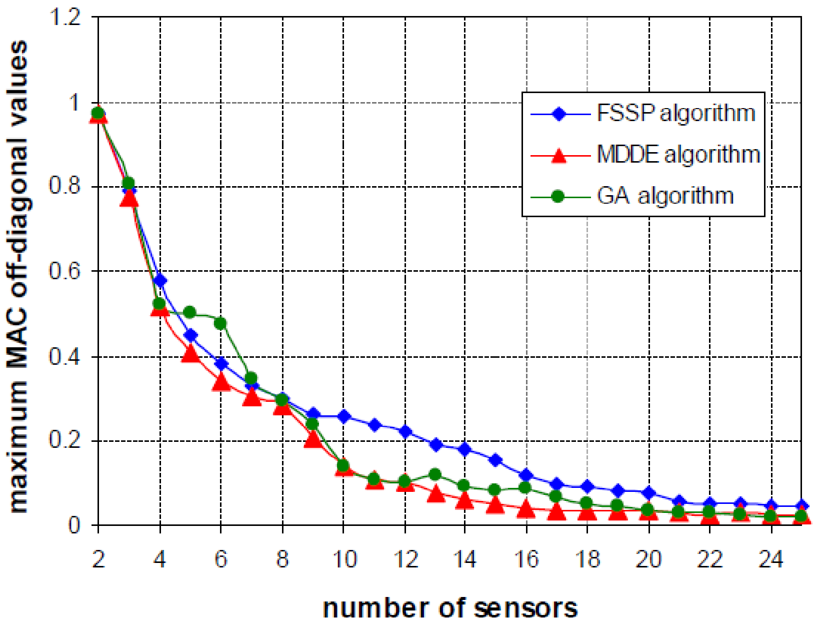

Figure 8: Diagram of changes in the MAC objective function based on the number of sensors for the 3S1S frame

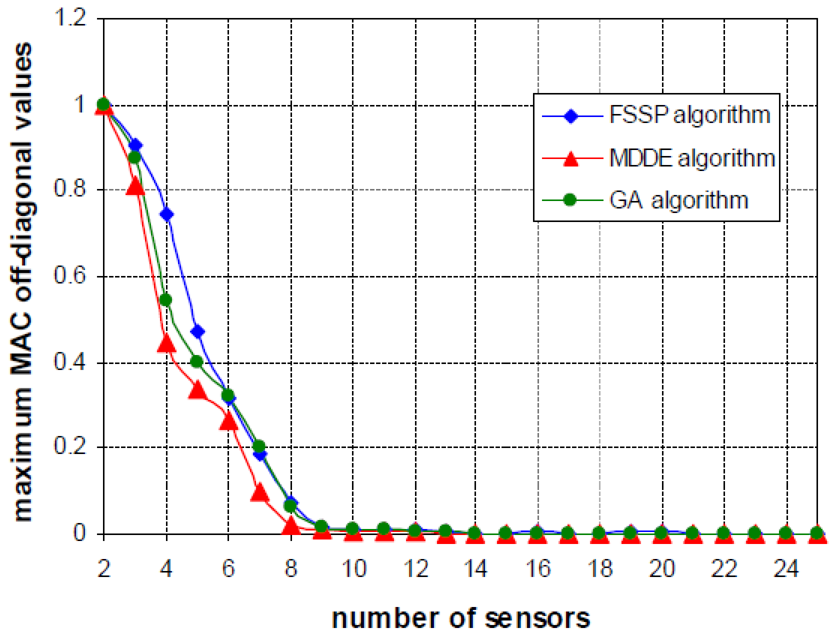

Figure 9: Graph of changes in MAC objective function based on the number of sensors for the 1S1S frame

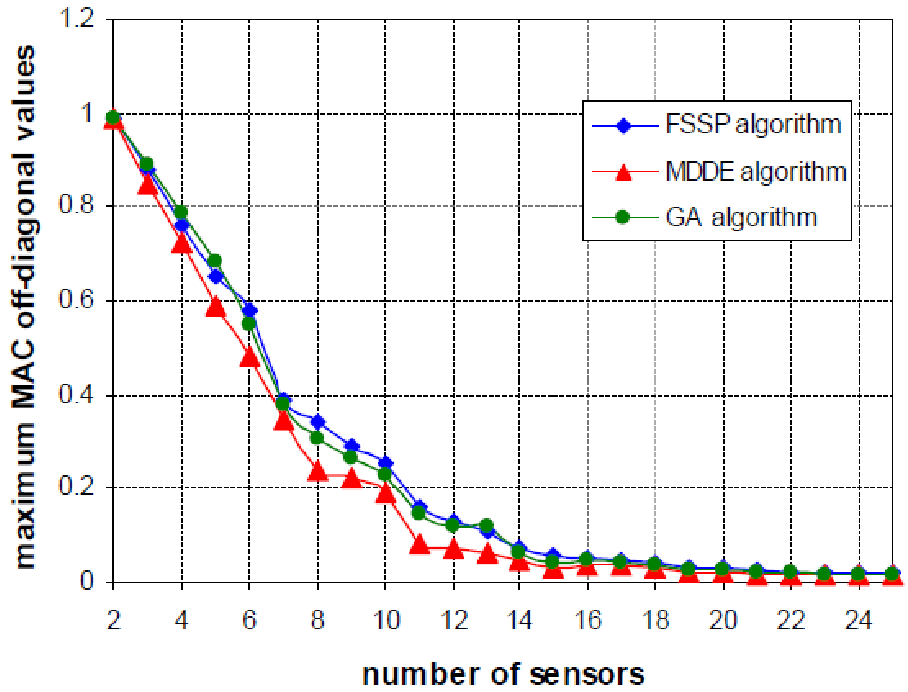

Figure 10: Graph of changes in MAC objective function based on the number of sensors for 2S2S frame

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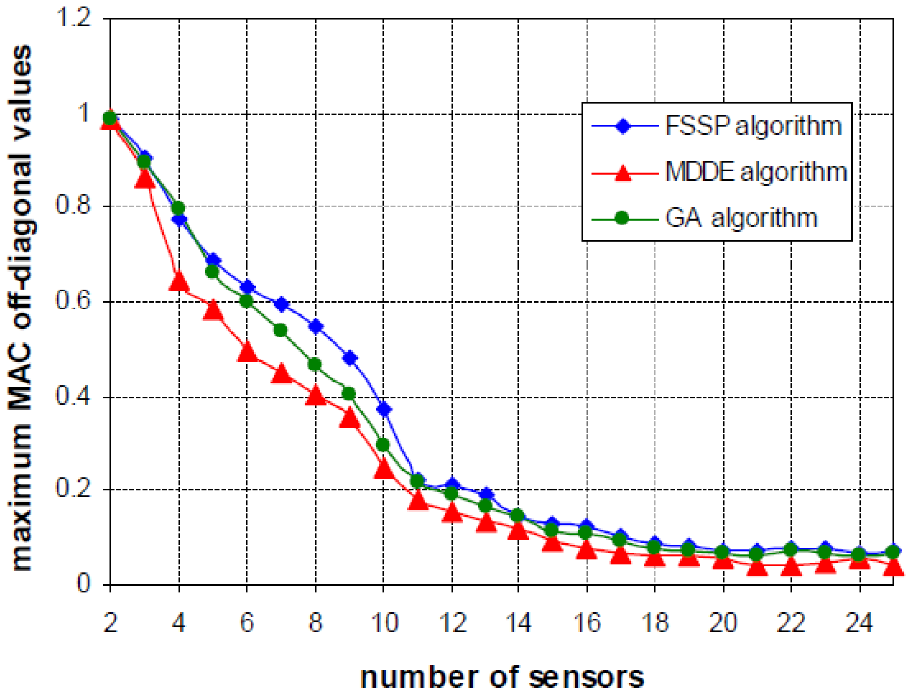

Figure 11: Graph of changes in MAC objective function based on the number of sensors for the 3S2S frame

Conclusion:

The research section on optimal sensor design is modeled linearly in the Appendix software for six two-dimensional CFS frames with different floors and openings. Modal analysis was performed on the models in the software, and the modal shape values were extracted from the degrees of structural freedom. The number of shape modes was selected based on the classes and openings of the frames in such a way as to cover the shape of the dominant modes and, at the same time, not be selected so large as to complicate the optimization problem. Degrees of freedom nominated for sensing included degrees of transitional freedom in which sensing was possible. The objective functions were introduced as the determinants of the Fisher information matrix and the largest non-diametric derivative of the Modi confidence measurement matrix, or MAC. Finally, the responses were compared with two methods of sequential sensor addition, or FSSP, and the Advanced Genetic Algorithm, or IGA. This optimization and comparison in frames are as follows: The following results can be extracted separately for each frame.

The proposed discrete differential evolution algorithm has been more efficient among the various methods of optimizing the location and number of sensors, such as the genetic algorithm and sequentially increasing the sensor. Thus, the objective function value obtained from the proposed method was more appropriate in a fixed number of sensors than the objective function values obtained from the other two methods.

The locations obtained from the proposed method as the optimal location of the sensors are different from the optimal locations obtained from the two comparative methods.

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The graph of changes in the value of the MAC objective function based on the number of sensors shows that the chart finds an almost constant value at a certain number of sensors. No decrease in the value of the objective function is observed. That number of sensors can be selected as the optimal number resulting from the MAC function's aim.

The optimal number of sensors obtained from two consecutive methods of adding a sensor and an advanced genetic algorithm is greater than the optimal number of sensors obtained from the improved discrete differential evolution algorithm. This means incurring more costs in the structural health monitoring process if older methods are used. As a result, the optimal number can be reduced, and more information can be obtained using the proposed method.

The graph of changes in the value of the FIM objective function based on the number of sensors shows that by increasing the number of sensors, the graph still maintains its upward trend, which is consistent with the principle of the Fisher information matrix. The more sensors, the more information received and the larger the value of the objective function. As a result, this method's optimal number of sensors should be determined based on economic considerations or equal to the optimal number of sensors obtained from the MAC target function.

Results of fault detection using neural network hierarchy

The optimal locations of the obtained sensors were used to detect the damage, and the damage detection based on the recorded information was successful. The results of this section are as follows:

Combining two failure indicators based on the reduced frequency response function and the accelerated response based on the reduced reduction time gives far better reactions than these failure indicators alone. The two failure indicators mentioned were proposed in the neural network hierarchy for the first time.

Doing damage detection in two steps gives accurate and efficient results. In this way, first, while optimizing the location and number of sensors, the number and locations of the structure from which information should be extracted are determined. Damage is then identified using the data extracted from them.

In identifying damage, perturbation is effective and can not be ignored. Although the proposed method is resistant to concern and has an increasing percentage of worry, the answers still have acceptable accuracy. However, according to the obtained results, it can be seen that when the share of perturbation increases, the accuracy of the method decreases, although this reduction is allowed within limits.

This study introduced the acceleration response failure index based on the reduced time as input to the neural network for the first time. The results of this index show that the responses of this index are far better than the response of the failure index of the reduced frequency response function. This improvement is especially noticeable at high turbulence percentages.

Performing fault detection by considering multiple faults in the braces shows that the proposed method is applicable even for various faults and can detect even the percentage of minor faults.

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Reference:

[1] Song J-H, Lee E-T, Eun H-C. Optimal sensor placement through expansion of static strain measurements to static displacements. International Journal of Distributed Sensor Networks. January 2021. doi:10.1177/1550147721991712

[2] Li, Z., J. Tang, and Q. Li, Optimal sensor locations for structural vibration measurements. Applied Acoustics, 2004. 65(8): p. 807-818.

[3] Doubling, S.W., C.R. Farrar, and M.B. Prime, A summary review of vibration-based damage identification methods. Shock and vibration digest, 1998. 30(2): p.91-105.

[4] Kim, J.-T. and N. Stubbs Crack detection in beam-type structures using frequency data. Journal of Sound and Vibration, 2003. 259(1): p. 145-160.

[5] Ndambi, J.-M., J. Vantomme, and K. Harri, Damage assessment in reinforced concrete beams using eigenfrequencies and mode shape derivatives. Engineering Structures, 2002. 24(4): p. 501-515.

[6] Garesci, F., L. Catalano, and F. Petrone, Experimental results of a damage detection methodology using variations in modal parameters. Experimental mechanics, 2006. 46(4): p. 441-451.

[7] Kim, J.-T., et al., Damage identification in beam-type structures: frequency-based method vs. mode-shape-based method. Engineering structures, 2003. 25(1): p. 57- 67.

[8] Salane, H. and J. Baldwin Jr, Identification of modal properties of bridges. Journal of Structural Engineering, 1990. 116(7): p. 2008-2021.

[9] Razak, H.A. and F. Choi, The effect of corrosion on the natural frequency and modal damping of reinforced concrete beams. Engineering structures, 2001. 23(9): p. 1126-1133.

[10] Choi, F., Assessment of the structural integrity of bridges using dynamic approaches. 2007, Ph.D. Thesis, Faculty of Engineering, University of Technology Sydney, Australia.

[11] Ching, J. and J. Beck, Bayesian analysis of the Phase II IASC-ASCE structural health monitoring experimental benchmark data. Journal of Engineering Mechanics, 2004. 130(10): p. 1233-1244.

[12] Choi, S., S. Park, and N. Stubbs, Nondestructive damage detection in structures using changes in compliance. International Journal of Solids and Structures, 2005. 42(15): p. 4494-4513.

[13] Hu, C. and M.T. Afzal, *A statistical algorithm for comparing mode shapes of vibration testing before and after damage in timbers.* Journal of Wood Science, 2006. **52**(4): p. 348-352.

[14] Unger, J.F., A. Teughels, and G. De Roeck, *Damage detection of a prestressed concrete beam using modal strains.* Journal of Structural Engineering, 2005. **131**(9): p. 1456-1463.

[15] . Hamey, C.S., et al., *Experimental damage identification of carbon/epoxy composite beams using curvature mode shapes.* Structural Health Monitoring, 2004. **3**(4): p. 333-353.

[16] Patjawit, A. and W. Kanok-Nukulchai, *Health monitoring of highway bridges based on a Global Flexibility Index.* Engineering Structures, 2005. **27**(9): p. 1385- 1391.

[17] Choi, F. et al., *Applying the modified damage index method to timber beams.* Engineering structures, 2008. **30**(4): p. 1124-1145.