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MATLAB SIMULATION OF QUADCOPTER DYNAMICS AND PID ATTITUDE CONTROLLER

T.A. Safarov [1. [0000-0002-6986-673X]]

[1] Azerbaijan State Oil and Industry University, Azerbaijan
AZ 1010, Azerbaijan, Baku, Azadlig avenue 20
seferov.tural96@gmail.com

Abstract. Due to the ever-growing popularity of quadcopters, they are used in various fields such as surveillance, military, surveillance and courier services and are of great importance. Ideal handling of quadcopters is essential for safe maneuvering and high precision flight performance. This research provides simulation of quadcopter control in MATLAB software, development and review of various control principles.

Dynamic development in unmanned aerial vehicles (UAVs) has led to the rapid spread of quadcopters in many fields, from airspace control to courier services. This research study investigates the Proportional-Integral-Derivative (PID) attitude controller by simulating the quadcopter dynamic object in MATLAB software. The heart of this research is to create and develop a PID attitude controller, which is a critical component for accurate control of quadcopter.

Key words. Quadcopter, UAV (Unmanned Aerial Vehicle), PID (Proportional-Integral-Derivative) control, attitude control, aerodynamic considerations, flight dynamics

Nomenclature.

General Symbols	
t	Time (s)
u	Control input vector
x	State vector
y	Output vector
Quadcopter Configuration	
m	Mass of the quadcopter (kg)
I_{xx}, I_{yy}, I_{zz}	Moments of inertia around the body-fixed axes (x, y, z) (kg·m²)
L	Distance from mass center of quadcopter to the motor (m)
b	Drag coefficient
g	Gravitational acceleration vector (m/s²)
F_{thrust}	Total thrust force (generated by all motors)
M_{drag}	Total drag moment (due to aerodynamic effects)
Dynamics and Kinematics	
v	Linear velocity vector (body frame) (m/s)

w	Angular velocity vector (body frame) (rad/s)
u, v, w	Linear velocity components (body frame) (m/s)
p, q, r	Angular velocity components (body frame) (rad/s)
Attitude and Rotation	
ϕ, θ, ψ	Roll, Pitch, Yaw angles (rad)
R	Rotation matrix representing the transformation between frames
$\dot{\phi}, \dot{\theta}, \dot{\psi}$	Euler rates (rad/s)
$R_z\phi, R_y\theta, R_z\psi$	Individual rotation matrices for yaw, pitch, and roll
Control and PID	
K_p, K_i, K_d	Proportional, integral, and derivative gains in PID control
e_ϕ, e_θ, e_ψ	Attitude errors (roll, pitch, yaw)
M_ϕ, M_θ, M_ψ	Moments applied around the quadcopter's axes
Coriolis Effect	
$[w]_\times$	Skew-symmetric matrix associated with angular velocity
w	Angular velocity vector in body frame (rad/s)
V_x, V_y, V_z	Linear velocities in inertial frame (m/s)
Equations of Motion	
$\dot{p}, \dot{q}, \dot{r}$	Time derivatives of angular rates (rad/s²)
M_ϕ, M_θ, M_ψ	Moments manufactured by roll, pitch, and yaw control inputs

Configuration of Quadcopter Airframe. We split the four motors clockwise, starting from the front arm. A propeller suitable for the direction of rotation of each motor must be used. For a high result, it is also important to choose the right propeller, and for this the specification of the motors should be examined.

Motor	Location	Rotating direction	Notes
1	Front	Counter-clockwise	$O_1 = +1, \xi_1 = 0^\circ$
2	Left	Clockwise	$O_2 = -1, \xi_2 = 270^\circ$
3	Back	Counter-clockwise	$O_3 = +1, \xi_3 = 180^\circ$
4	Right	Clockwise	$O_4 = -1, \xi_4 = 90^\circ$

Specifications . We enter some basic specifications to be used in the simulation environments.

Quadcopter Airframe	
Frame	Carbon fiber. X-shaped configuration
Dimensions	Diagonal motor-to-motor span: 450 mm
Weight	1.25 kg (including battery)
Arm Length	0.256 m
Motor to Motor Distance	Output vector
Propellers	10x4.5 inch, two clockwise (CW) and two counterclockwise (CCW) rotation
Battery	3S LiPo, 11.1V, 2200mAh
Motors	
Model	XYZMotor 2206
KV Rating	2300 RPM/V

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Max Thrust	600 g per motor
ESC Compatibility	20A Electronic Speed Controllers (ESCs)
Electronic Speed Controllers (ESCs)	
Model	ESCMaster Pro 20A
Voltage	2-4S LiPo
BEC Output	5V, 1A
Communication Protocol	PWM
Sensors and Actuators	
IMU	MPU-6050 6-DOF (Gyroscope and Accelerometer)
Communication Protocol	I2C
Control Surfaces	None (fully quadcopter configuration)
Flight Controller	
Model	FlightBrain 3.1
Processor	ARM Cortex-M4
Clock Speed	168 MHz
Communication Protocols	PWM, I2C, SPI
PID Controller Gains (Initial Values)	
Roll	$K_p = 4.0, K_i = 0.1, K_p = 0.05$
Pitch	$K_p = 4.2, K_i = 0.15, K_p = 0.06$
Yaw	$K_p = 3.5, K_i = 0.08, K_p = 0.03$
x moment of inertia	$0.0232\ \mathrm{kg} \cdot m^2$
y moment of inertia,	$0.0232\ \mathrm{kg} \cdot m^2$
z moment of inertia	$0.0468\ \mathrm{kg} \cdot m^2$

Dynamic model of quadcopter. State variables.

x	Position along the quadcopter's x-axis (m)
y	Position along the quadcopter's y-axis (m)
z	Position along the quadcopter's z-axis (m)
ϕ	Roll angle (rad)
θ	Pitch angle (rad)
ψ	Yaw angle (rad)
v_x	Velocity along the quadcopter's x -axis (m/s)
v_y	Velocity along the quadcopter's y -axis (m/s)
v_z	Velocity along the quadcopter's z -axis (m/s)
p	Angular velocity around the quadcopter's x -axis (rad/s)
q	Angular velocity around the quadcopter's y -axis (rad/s)
r	Angular velocity around the quadcopter's z -axis (rad/s)

Control variables. The four control inputs, which represent the total thrust and torques on axis of control, are listed below.

$$\vec{u} = [u_1, u_2, u_3, u_4]^T = [T_{\sum}, M_1, M_2, M_3]^T$$

Equations of Motion. We can describe motion equations for the quadcopter using a set of differential equations that relate the state variables to their derivatives:

1. Translational Motion.

$$\dot{x} = v_x$$

$$\dot{y} = v_y$$

$$\dot{z} = v_z$$

Based on their velocities, these equations show us how the quadcopter's positions vary over time.

2. Rotation Motion.

$$\dot{\phi} = p + q \cdot \tan\theta \cdot \sin\phi$$

$$\dot{\theta} = q \cdot \cos\phi - r \cdot \sin\phi$$

$$\dot{\psi} = \frac{q \cdot \sin\phi + r \cdot \cos\phi}{\cos\theta}$$

Based on its angular velocities, these equations show how orientation angles of quadcopters vary over time.

3. Translational Velocity:

$$\dot{v_x} = g \cdot \theta - \frac{T}{m} \cdot \sin\theta$$

$$\dot{v_y} = -g \cdot \phi + \frac{T}{m} \cdot \cos\theta \cdot \sin\phi$$

$$\dot{v_z} = -g + \frac{T}{m} \cdot \cos\theta \cdot \cos\phi$$

According to thrust, gravity, and the quadcopter's orientation, these equations show how the quadcopter's velocities varies over time.

4. Rotational Velocity:

$$\dot{p} = \frac{I_{yy} - I_{zz}}{I_{xx}} \cdot q \cdot r - \frac{M_\phi}{I_{xx}}$$

$$\dot{q} = \frac{I_{zz} - I_{xx}}{I_{yy}} \cdot p \cdot r + \frac{M_\theta}{I_{yy}}$$

$$\dot{r} = \frac{I_{xx} - I_{yy}}{I_{zz}} \cdot p \cdot q + \frac{M_\psi}{I_{zz}}$$

These equations explain how moments and moments of inertia affect the quadcopter's angular velocities as they varies over time.

Aerodynamic Considerations. Rotor Thrust and Torque. The force generated by the rotors of the quadcopter serves as lift as well as the main source of control. Different variables can affect the thrust of each rotor: rotor speed, blade geometry, angle of attack, and airfoil form. The quadcopter's capacity to take off and transport payloads is determined by the combined thrust from all of its rotors.

Thrust of a Single Rotor: $T_i = k \cdot \omega_i^2$

- T_i is the thrust of the i -th rotor.
- k is a constant that represents the rotor characteristics.
- ω_i is the angular velocity of the i -th rotor.

Roll Torque from Rotors: $M_\phi = d \cdot (T_1 - T_2 + T_3 - T_4)$

- M_ϕ is the roll torque.
- d is the distance from the center of the quadcopter to each rotor.

Drag and Air Resistance. Whereas drag is for the most part less critical for quadcopters compared to airplanes, it still influences their flight effectiveness. the quadcopter's frontal range, shape, and speed are the main characteristics that cause drag.

Drag Force: $F_{drag} = \frac{1}{2} \cdot C_d \cdot \rho \cdot A \cdot k \cdot V^2$

- C_d is the drag coefficient.
- ρ is the air density.
- A is the effective frontal area.
- V is the velocity of the quadcopter.

Yaw and Roll Interactions. The quadcopter's roll and yaw characteristics are impacted by the precise force produced by the pivoting rotors. Turning around the vertical hub creates opposing torques that can affect soundness. Appropriate control algorithms and adjustments are required to manage these interactions.

Roll Moment due to Yawing: $M_\phi = -k_{yaw} \cdot \dot{\psi}$

- k_{yaw} is a constant represents to yaw-roll interaction.
- $\dot{\psi}$ is the yaw rate.

Thrust and Moment Distributions for a Quadcopter. Thrust Distribution. A quadcopter's total thrust is the sum of the thrust of the individual rotors: $T = \sum_{i=1}^{4} T_i = k_T \sum_{i=1}^{4} \Omega_i^2$

Moment Distribution. The moments of axes of the quadcopter are the outcome of differential thrusts of rotors. They can help to the quadcopter's rotation and control.

$$M_\phi = d \cdot (T_1 - T_2 + T_3 - T_4)$$

$$M_\theta = d \cdot (T_1 + T_2 - T_3 - T_4)$$

$$M_\psi = c \cdot (T_{CW} - T_{CCW})$$

- $M_\phi$is the roll moment.
- M_θ is the pitch moment.
- M_ψ is the yaw moment.
- d is the distance from the center of the quadcopter to each rotor.
- T_i is the thrust produced by the i-th rotor.
- c is a constant related to the rotor's distance from the quadcopter's center and the rotor's moment arm.
- T_{CW} is the thrust produced by the clockwise spinning rotors.
- T_{CCW} is the thrust produced by the counterclockwise spinning rotors.

Equations of Motion. Translational Motion.

$$m \cdot \dot{v_x} = -T \cdot \sin\theta - D_{drag} \cdot v_x$$

$$m \cdot \dot{v_y} = T \cdot \cos\theta \cdot \sin\phi - D_{drag} \cdot v_y$$

$$m \cdot \dot{v_z} = T \cdot \cos\theta \cdot \cos\phi - m \cdot g$$

- m is the mass of the quadcopter.
- $\dot{v_x}, \dot{v_y}$ and $\dot{v_z}$ are the accelerations in the x, y and z directions, respectively.
- T is the total thrust.
- θ is the pitch angle.
- ϕ is the roll angle.
- D_{drag} represents the drag force.

Rotational Motion.

$$\dot{p} = \frac{I_{yy} - I_{zz}}{I_{xx}} \cdot q \cdot r - \frac{M_\phi}{I_{xx}}$$

$$\dot{q} = \frac{I_{zz} - I_{xx}}{I_{yy}} \cdot p \cdot r + \frac{M_\theta}{I_{yy}}$$

$$\dot{r} = \frac{I_{xx} - I_{yy}}{I_{zz}} \cdot p \cdot q + \frac{M_\psi}{I_{zz}}$$

- m is the mass of the quadcopter.
- I_{xx}, I_{yy} and I_{zz} are the moments of inertia around the x, y and z axes, respectively.
- $\dot{p}, \dot{q}$ and $\dot{r}$ are the angular accelerations around the x, y and z axes, respectively.
- p, q and r r are the angular velocities around the x, y and z axes, respectively.

- M_ϕ, M_θ and M_ψ are the moments produced by roll, pitch, and yaw control inputs, respectively.

Attitude control of a quadcopter. The quadcopter's roll, pitch and yaw angles must be controlled and stable to perform the desired movements. The process of linearization involves building a linear model of the nonlinear dynamics of a system, such as a quadrocopter, around an operating point. Using this linear model simplifies control design and analysis. Here we describe attitude control in detail and show how the dynamics of a quadcopter can be linearized.

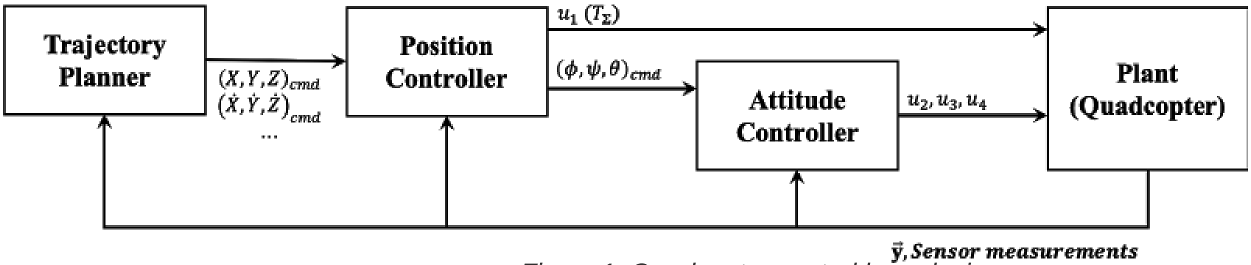

Figure 1. Quadcopter control loop design

Linearization. A general nonlinear equation of motion for a quadcopter is: $\dot{x} = f(x,u)$

x is the state vector representing the quadcopter's state variables.

u is the control input vector.

$f(\cdot)$ is the nonlinear function that describes the dynamics.

Taylor series expansions can be used to linearize these equations around the operating point:

$$\dot{x} = f(x_0,u_0) + \frac{\partial f}{\partial x}\bigg|_{x_0,u_0} (x - x_0)\frac{\partial f}{\partial u}\bigg|_{x_0,u_0} (u - u_0) + O(\|x - x_0\|^2, \|u - u_0\|^2)^2$$

Higher order terms can be ignored to linearize the equation as

$$\dot{x} \approx A(x - x_0) + B(u - u_0)$$

$A = \frac{\partial f}{\partial x}\Big|_{x_0,u_0}$ is the state matrix.

$B = \frac{\partial f}{\partial u}\Big|_{x_0,u_0}$ is the control matrix.

This linearization procedure allows us to study stability and develop control strategies for the quadcopter dynamics using linear control theory techniques.

For the particular use of linearization in quadcopter dynamics, we can define equations of motion for roll, pitch and yaw velocities as functions of control inputs and state variables. The state and control matrices are then obtained by computing the partial derivatives and evaluating them at the desired operating point. Here is a simplified way to linearize the quadcopter roll rate dynamics equation:

Nonlinear Equation: $\dot{p} = \frac{I_{yy}-I_{zz}}{I_{xx}} \cdot q \cdot r - \frac{M_\phi}{I_{xx}} - (2 \cdot m \cdot r \cdot V_y + (I_{yy} - I_{zz}) \cdot q \cdot w)$

Linearized Equation: $\Delta\dot{p} \approx A_p \Delta p + B_p \Delta M_\phi$

- $\Delta\dot{p}$ is the change in p around the operating point.
- Δp is the change in p around the operating point.
- A_p and B_p are obtained by computing the partial derivatives with respect to state variables and control inputs.

PID Controller Design. Controller design for quadcopter attitude control often uses more advanced techniques such as PID (Proportional-Integral-Derivative) control and LQR (Linear Quadratic Regulator).

We have to set the desired target value as the center of gravity for the quadrocopter flight attitude (roll, pitch and yaw angles). The moments (M_ϕ, M_θ and M_ψ) applied around the quadcopter's axis is called the control input. The deviation between the desired target values ($\phi_{des}, \theta_{des}, \psi_{des}$) and the actual angles (ϕ, θ, ψ) should also be included in the definition of position error.

- Roll error: $e_\phi = \phi_{des} - \phi$
- Pitch error: $e_\theta = \theta_{des} - \theta$
- Yaw error: $e_\psi = \psi_{des} - \psi$

Based on the error and its time derivative, a PID control law computes the control input.

$$u = K_p \cdot e + K_i \cdot \int e \, dt + K_d \cdot \frac{de}{dt}$$

- u is the control input (moment).
- K_p, K_i, K_d are the proportional, integral, and derivative gains, respectively.
- e is the error.
- $\int e \, dt$ is the integral of the error with respect to time.
- $\frac{de}{dt}$ is the derivative of the error with respect to time.

Apply the PID control law to compute the control inputs (M_ϕ, M_θ and M_ψ) for each axis (roll, pitch, yaw).

$$M_\phi = K_{p\phi} \cdot e_\phi + K_{i\phi} \cdot \int e_\phi \, dt + K_{d\phi} \cdot \frac{de_\phi}{dt}$$

$$M_\theta = K_{p\theta} \cdot e_\theta + K_{i\theta} \cdot \int e_\theta \, dt + K_{d\theta} \cdot \frac{de_\theta}{dt}$$

$$M_{\psi} = K_{p\psi} \cdot e_{\psi} + K_{i\psi} \cdot \int e_{\psi}\, dt + K_{d\psi} \cdot \frac{de_{\psi}}{dt}$$

Matlab Simulation.

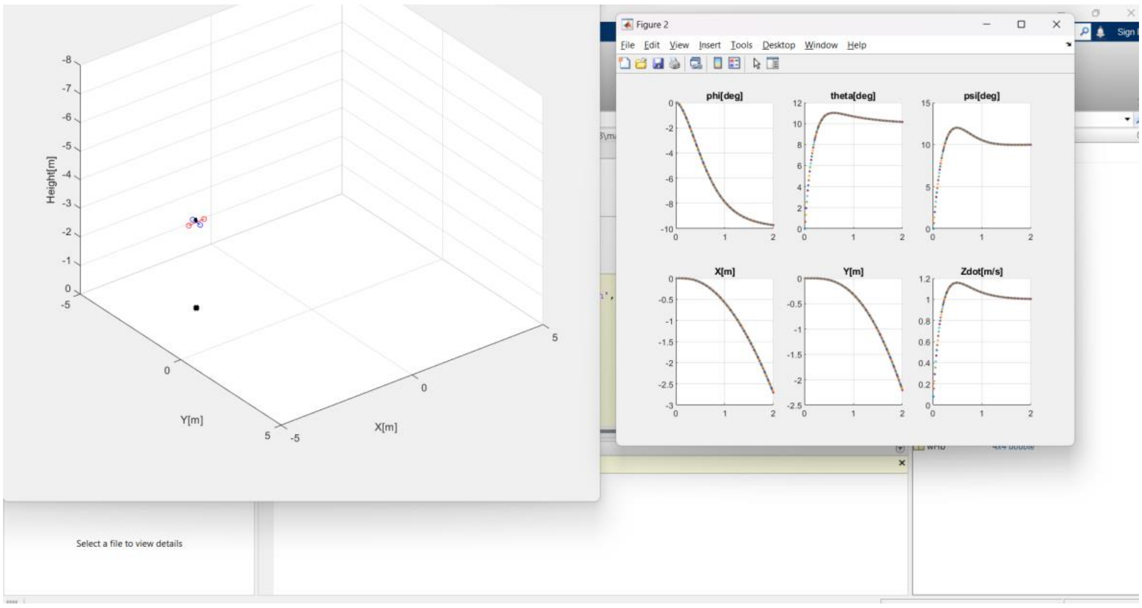

Figure 2. MATLAB simulation of Quadcopter control

Using a streamlined model, the provided MATLAB code illustrates the simulation of quadcopter dynamics and control. The program creates a quadcopter object with the required settings, applies PID attitude control, and updates the quadcopter's status over the course of the simulation.

Conclusions. The 3D graphic offers a clear illustration of the quadcopter's ability to manage its orientation, attitude, and movement in a 3D space. The data plots also show the temporal evolution of important parameters including roll, pitch, yaw, and location.

The quadcopter's attitude is controlled and stabilized via the PID attitude control technology. By examining how the quadcopter reacts to the inputs commanding its attitude, the efficiency of the control strategy can be assessed.

Future work could involve incorporating additional control algorithms, such as more advanced control strategies (e.g., Model Predictive Control), trajectory tracking, or autonomous navigation. Furthermore, optimizing the control gains and evaluating the quadcopter's performance under various scenarios would contribute to a deeper analysis of the control system.

In summary, the provided MATLAB simulation offers an entry point for exploring quadcopter dynamics and control. The code's visualization and data plots provide valuable insights into the quadcopter's behavior, and the simulation environment can serve as a stepping stone for more advanced studies in UAV dynamics and control.

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