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Proportional Integral Accelerator Controller Design and Theoretical Investigation of Disturbance Rejection Performance

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Abstract. In this study, Proportional Integral Accelerator (PIA) controller parameters are obtained using Random Search (RS) optimization algorithm by compromising between Reference to Disturbance Rate (RDR) criteria and control error. Thus, robustness performance of PIA controller against the input disturbance is investigated. First, the robustness of the system controlled by the PIA against input disturbance is theoretically demonstrated. Then, the performance analysis of the PIA control system whose parameters obtained by RS optimization is examined with application examples in Matlab/Simulink environment.

Keywords. PIA controller, RDR criteria, Disturbance rejection, Controller optimization.

1. Introduction

PI, PD and PID controllers are highly preferred in the industry due to their robust performance and simple structure [1-4]. A PIDA controller, which is recommended by Jung and Dorf in 1996 by adding the term acceleration (A) to the PID controller for the control of high-order systems, have also been used in some applications in recent years [5-6]. As in all controllers, environmental disturbances and internal disturbances, generated by the system, seriously affect the performance of the PIDA controllers. Thus determining the effect of acceleration term, which is added to PID-type controllers to obtain PIDA-type, on disturbance rejection performances can be an important research topic. Until now, various studies have been conducted on the disturbance rejection capacity of PI and PID controllers and many suggestions have been put forward [7-13]. It has been suggested that determining the disturbance rejection capacity with the RDR index in controller design is important for the disturbance rejection performance of control systems [8-10]. The RDR index has been applied in some studies in recent years in controller design [11-14] and in the investigation of disturbance rejection performance [12].

The controller's performances are significantly affected by environmental and internal disturbances. In order to minimize these effects, two methods are generally applied in disturbance rejection performance of control systems. The first are open approximation methods (adding additional expressions and additional blocks such as filters, noise and state observers, noise estimators, adaptive state feedback controllers) [15-18]. The second is closed approximation methods (stability, pole placement methods, sensitivity function limitation methods, and the design methods based on increasing RDR index performance) [18-21].

In this study, a PIA controller design method is proposed using RS algorithm in a closed loop feedback control system to improve set-point control performance while maximizing the RDR index.

Generally the main goal of the design problems is to determine the coefficients of controllers that ensure both disturbance rejection performance and set-point control performance to be acceptably good. The results and efficiency of this proposed design method were determined via design examples and the disturbance rejection capacity and resistance to input disturbances of the control systems were identified.

2. Theoretical Background

2.1. Random Search Algorithms

In order to exhibit their desired application performance for systems, optimal design is required. For this reason, heuristic optimization algorithms are widely used in controller designs nowadays. The RS algorithm is a low computational stochastic search method compared to the random search method and can give very good results in some problems. Therefore, it has an interesting feature for practitioners and theorists. RS methods have some advantages over other search methods, such as programming ease, computational efficiency, and being applicable to almost any function. [22].

2.2. PIA Controller Structure

It is claimed that PIDA controllers, which are obtained by adding acceleration term (A) to the PID controller, respond more effectively in higher order systems [3]. From this inference, the PIA controller can be obtained by adding acceleration term (A) to the PI controller. The transfer function of the PIA controller can be expressed as follows.

$$C_{PIA}(s) = K_P + \frac{K_i}{s} + K_a s^2 \tag{1}$$

Recently, studies have been conducted to examine the performance of PIA controllers. For example, in a control application performed in a fixed speed wind turbine plant, classical PI and PIA were designed by using optimization techniques and it was observed that PIA provided better performance [23-25].

2.3. RDR Analysis and Disturbance Rejection

Various disturbance rejection methods have been proposed for closed loop control systems up to date [17, 21]. For example; Visioli proposed load disturbance rejection performance PID controllers [26], Vrancic has showed an improvement of PID controller's disturbance rejection by amplitude optimum method [11], Chen proposed PI/PID design methods and disturbance rejection performances based on direct synthesis [13].

In closed loop control systems, the RDR criterion expresses the input disturbance rejection capacity depending on the angular frequency. Optimization methods have been applied in RDR computations to improve the disturbance rejection performance of the closed loop control system [8-10]. In this study, the disturbance rejection capacity of the closed loop control system controlled by the PIA controller is determined using the RDR criterion.

RDR analysis is defined by the ratio of the energy of the reference signal at the system output to the energy of the disturbance signal. RDR >> 1 indicates that the control system has good disturbance rejection performance, while RDR << 1 indicates that the control system's disturbance rejection performance is insufficient [8-10]. In a good controller design, the optimization process should ensure that the RDR criterion is high, but the control error is low. Therefore, in this study, optimization process, using RS algorithm, has been made by providing a compromise between the RDR criterion and the control error. The optimization process is managed according to the consensus curve, trying to reach an optimal performance point on the curve. In the RDR method, it is assumed that the energy of the reference signal ($|r(j\omega)|^2$) and the energies of the disturbance signal ($|d(j\omega)|^2$) are equal [14]. In this case RDR criterion can be expressed as,

$$RDR = \frac{|P_r(j\omega)|^2}{|P_d(j\omega)|^2} = |C(jw)|^2 \quad (2)$$

where, $P_r(j\omega)$ is the closed loop transfer function of the control system with reference input, $P_d(j\omega)$ is the closed loop transfer function of the control system with disturbance signal input and $C(j\omega)$ is the transfer function of the controller [10]. Because of its very high numerical values, it is recommended to express the RDR index in decibels as follows,

$$RDR_{dB}(\omega) = 20\ log|C(j\omega)| \quad (3)$$

3. Obtaining RDR Criteria for PIA Controller

One can obtain following equations by using s=jω in the transfer function of PIA controller in Eq.1,

$$C(j\omega) = K_p + \frac{K_i}{j\omega} + K_a(j\omega)^2 \quad (4)$$

If the necessary arrangements are made, the amplitude of the PIA controller is written as,

$$|C(j\omega)| = \sqrt{\left(k_p - k_a\omega^2\right)^2 + \left(-\frac{k_i}{\omega}\right)^2} \quad (5)$$

Accordingly, RDR criteria for the PIA controller is,

$$RDR_{PIA}(\omega) = \left(k_p - k_a\omega^2\right)^2 + \left(\frac{k_i}{\omega}\right)^2 \quad (6)$$

where, k_p, k_i and k_a are gain coefficients of PIA controllers, In the design of a PIA controller, these 3 design parameters are optimally determined to achieve the desired control response. In Fig. 1, for the case that the design coefficients are 1 ($k_p = 1$, $k_i = 1$ ve $k_a = 1$), RDR performances of PIA and PI controller have been determined within the region $0 < \omega < 1000$ rad/s.

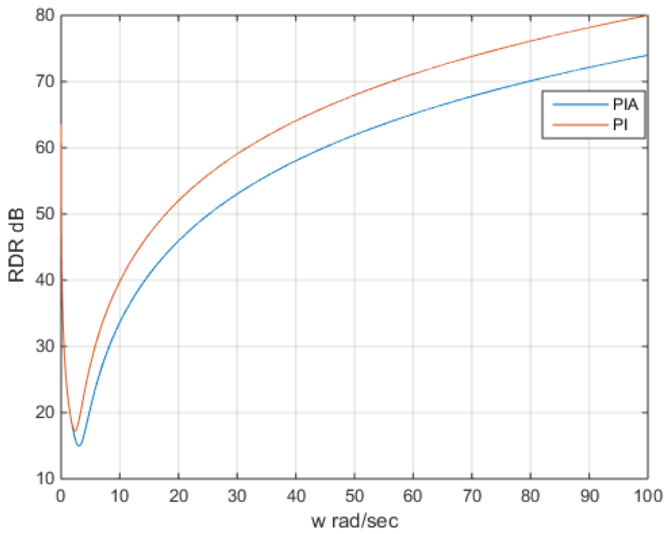

Fig. 1. RDR Spectra of the closed loop PIA and PI control system.

The figure shows that although the PIA controller shows lower RDR performance than the PI controller around 1 rad/s frequency, it generally exhibits better RDR performance in the mid and high frequency regions. However, it is seen that in case of a disturbance with a frequency of 1 rad/s, the system gets stronger at its output and decreases the RDR value.

4. PIA Controller Design with Increased Disturbance Rejection Performances Using RS Algorithm Based on RDR Index

In this section, in order to improve the disturbance rejection performance of the closed loop control system controlled by the PIA controller, a consensus curve based RS algorithm is applied for the optimal design of the parameters. As seen in Figure 2, a set-point filter $F(s)$ has been added to the unity feedback PIA control system. The pre-filter $F(s)$ is a first-order filter used to shape the reference input signal to filter out very high frequency components at the input [20]. Thus, the RDR of the control system can be further increased without impairing the unit step response, and it is ensured that both disturbance rejection and set-point control performances are improved.

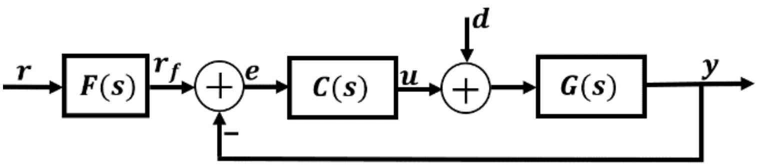

Fig. 2. PIA control system with pre-filter.

In this system, the disturbance signal (d) is assumed to be an external disturbance coming from outside to the controlled system and added to the control signal. The pre-filter function used in Fig. 2 is written as follows,

$$F(s) = \frac{a}{s+a} \tag{7}$$

$a = 1/\tau$ where τ is time constants. The unit step performance of the control system is evaluated with the square error sum as follows,

$$E = \frac{1}{T}\int_0^T e(t)^2 \cdot dt \tag{8}$$

Minimizing the amplitude of the control error (e) within a period of T and approximating the system output y(t) to the reference input r(t) is the main goal of the control system ($e(t) = r_f(t) - y(t)$). On the other hand, in order to increase the disturbance rejection performance of the control system, the minimum RDR requirement must be met for the RDR criterion of the PIA controller expressed in Eq.6. The primary purpose of this proposed control design method is to keep the system stable and to follow the set-point. Secondary objective is to increase the minimum RDR level of the controller under the condition that the primary objective is achieved. Otherwise, high disturbance rejection performance with low controllability is not desired for the control application. This situation raises the multi-objective optimization problem. This optimization problem is solved for the following constraint, which is the dynamic form of minimum RDR constraints.

$$\min\{10\log(RDR_{PIA}(w)\} \geq M(E), \tag{9}$$

$$M(E) = -\propto \log_{10} E,\ w \in [w_{min}, w_{max}] \tag{10}$$

Here $M(E)$ is the dynamic lower limit for the minimum RDR boundary. The decrease of E causes the minimum lower bound $M(E)$ to increase by a factor of $\propto$. Eq. 10 is called the consensus curve. Here the parameter $\propto$ is an acceptable minimum RDR and a logarithmic consensus coefficient used to meet a desired E value and can be determined in the form below,

$$\propto = \frac{\min(10\log(\mathrm{RDR_{PIA}}(w))}{\log_{10} E} \tag{11}$$

A curve can be drawn for the parameter $\propto$ of the consensus curve $M(E)$ [14]. This curve represents a dynamic lower bound for the minimum acceptable RDR in optimization and provides a compromise between the square error sum E in Eq. 8 and the RDR criterion.

The operation steps of the basic RS algorithm proposed to solve this optimization problem can be summarized as follows;

- Step 1: The initial values of the k_p, k_i and k_a controller parameters are set to any value that makes the system stable. A high value is set for the initial values of the filter parameter a and squared error parameter E_{min}.
- Step 2: New random candidate points are created in the search space with the recursive equations given below,

$$k_{pn} = k_p + (rand - 0.5)c_g; \quad k_{in} = k_i + (rand - 0.5)c_g;$$
$$k_{an} = k_a + (rand - 0.5)c_g\ ; \quad a_n = a + (rand - 0.5)c_f \tag{12}$$

- Step 3: To determine the operating frequency range $w \in [w_{min}, w_{max}]$ and $min\{RDR_{dB}\}$ constraint, the error function E is calculated for the unit step response.
- Step 4: Dynamic RDR boundary $M(E_{min}) = -\propto \log_{10} E_{min}$ is computed for minimum error E_{min}.
- Step 5:
  If $E < E_{min}$ and $min\{RDR_{dB}\} \geq M(E_{min})$
  Then
  $k_p = k_{pn},\ k_i = k_{in},\ k_a = k_{an},\ a = a_n$
  and update $E_{min} = E$
- Step 6: If E_{min} is sufficiently small or maximum iteration count is exceeded then, end optimization. Otherwise go to Step 2.

where, c_g and c_f are random values to alter the gain and filter coefficients respectively.

5. Application Examples

In PIA design problems, it can be predicted that using the RDR index as a design criterion will contribute to increase the disturbance rejection capacity of closed loop control systems. Because the use of RDR design constraint can increase the effectiveness of heuristic optimization algorithms as it narrows the search regions. This section shows examples of PIA controllers with disturbance inputs to validate theoretical information. These examples were carried out in Matlab Simulink environment. In optimization, random variation multipliers of parameters were taken as $c_g = 0.05$ ve c_f=0.2.

5.1. Example 1 :

A stable second order plant transfer function in Eq. 13 is considered. In order to design a PIA controller satisfying both good set-point control and improved disturbance rejection capacity.

$$\mathrm{G(s)} = 1/(\mathrm{s}^2 + 4\mathrm{s} + 3) \tag{13}$$

By applying the design steps for this system, the following PIA controller was obtained and the RDR performance was examined.

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$$C_{PIA_1}(s) = 13.065 + \frac{11.8690}{s} - 0.6179s^2 \quad (14)$$

Figure 3 shows the RDR spectrum for the designed PIA controller. It shows very high RDR performance especially in the zero and very low frequency region, and it indicates that the reference signal in this frequency region will be much more dominant at the control system output. At low values of angular frequency, RDR performance for frequency components appears to be minimal and disturbance rejection performance against environmental disturbances will be the worst. The RDR index rises again in the high frequency region, indicating that the control system, which is effective especially in the high frequency region, will be resistant to white noise. Figure 4 shows the variation of design parameters during the optimization of the PIA controller. Figure 5 shows the change of the control error during optimization. In this graph, the error value converges to zero and it is asymptotic, indicating that optimization has taken place and the control performance has been optimized by compromising with the minimum RDR lower limit value. When the optimization is completed, the control error E_{min} for $\min\{RDR_{dB}\} = 24.3026\ dB$ has been reduced to 0.003097. In Fig. 6, the unit step responses and disturbance rejection performances of the system in Eq. 13 are given for PIA and optimized PI controller in Matlab/Simulink in Eqs. 14 and 15, respectively.

$$C_{PI}(s) = 5.20 + \frac{4.5574}{s} \quad (15)$$

Here, at 20 seconds, the disturbance signal in the form of unit step was applied to the controlled system input and a unit step response was obtained for both control systems. It is clearly seen in the figures that the PIA controller can offer better control performance than the classical PI. These results show that the two degree of freedom closed loop PIA controller design with set-point filter can improve both the set-point control performance and the disturbance rejection performance together. It can be said that the consensus curve based RS algorithm implemented in this example performs well.

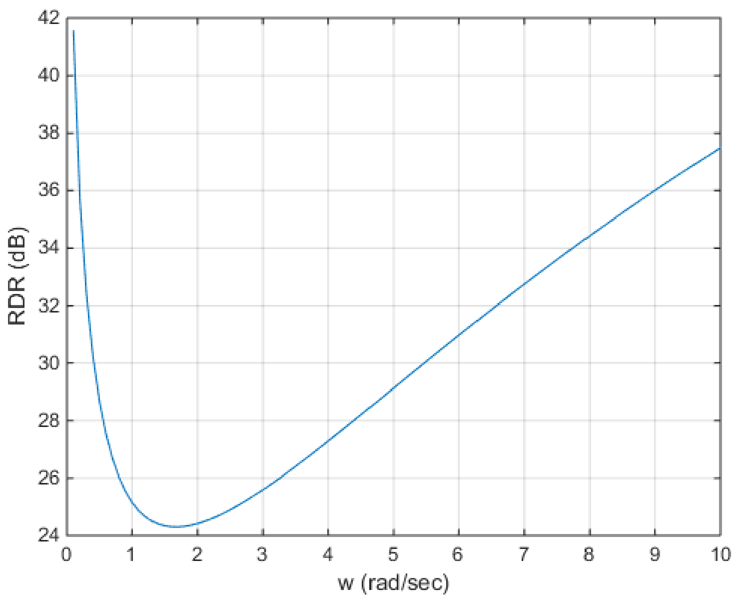

Fig. 3: RDR spectrum of the PIA controller.

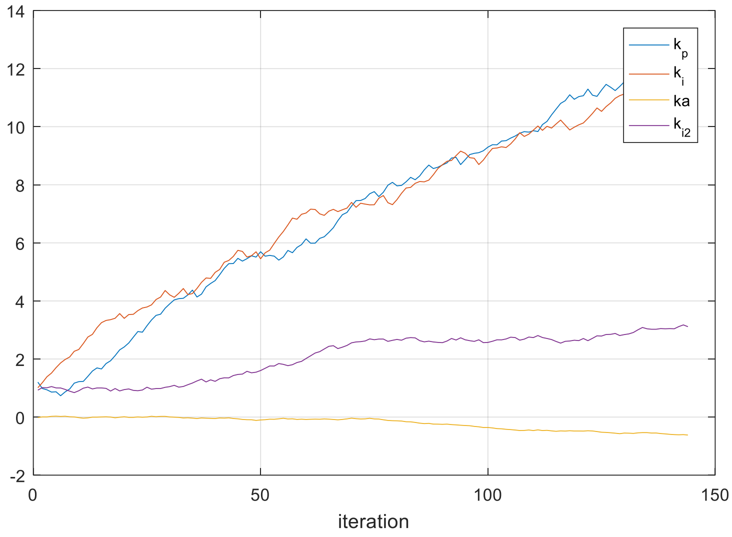

Fig. 4: Variation of design parameters during the optimization process.

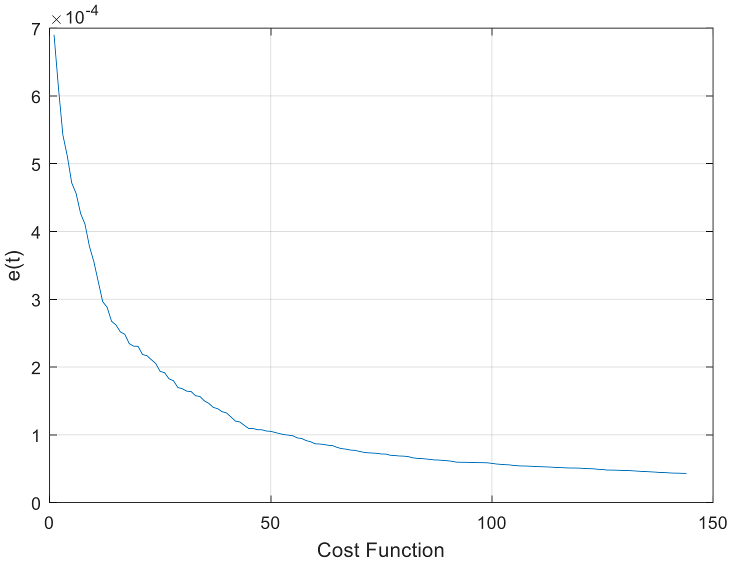

Fig. 5: Variation of E during optimization.

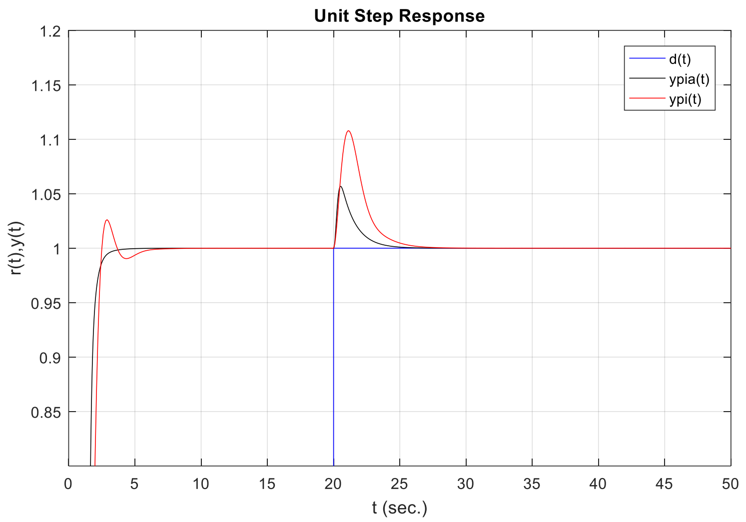

Fig. 6: Unit step response of PIA and PI control system.

5.2. Example 2:

Following stable first order plus dead time plant is considered;

$$G(s) = \frac{1}{9s+1} e^{-0.25s} \tag{16}$$

For this system, by applying the design steps given in section 4, the PIA controller in Eq. 17 was obtained by taking the RDR index into consideration. Then, the set-point control and disturbance rejection performance of the designed PIA controller were examined. As a result of the optimization, the control error is decreased to the value of $E_{min} = 0.006923$ for the value of $\min\{RDR_{dB}\} = 21.6984dB$.

$$C_{PIA_2}(s) = 12.1614 + \frac{4.3926}{s} - 0.0001s^2 \tag{17}$$

In addition, for the same system, optimized PI controller in Matlab / Simulink is given below.

$$C_{PI}(s) = 3.8074 + \frac{0.2379}{s} \tag{18}$$

In Figure 7, the unit step responses and disturbance rejection performances of the system in Eq. 16 controlled by the PIA in Eq. 17 and the PI in Eq. 18 are given.
As in the first example, the disturbance signal in the form of a unit step was applied to the controlled system input in the 20th second and a unit step response was obtained for both control systems. As seen in Figure 7, clearly it is seen that the PIA controller can offer better control performance compared to the classical PI. These results show that the first order time delay closed loop PIA control system design with set-point filter can improve both the set-point control performance and disturbance rejection performance together.

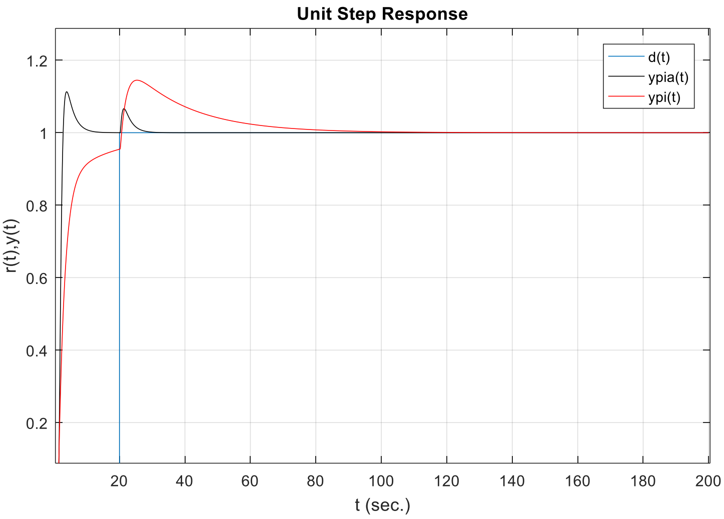

Fig. 7: Unit step response of PIA and PI control systems.

6. Conclusions

In this study, a PIA controller is designed using a multi-cost function RA algorithm based on RDR index analysis for a two degree of freedom closed loop system with a set-point filter. The performances of the designed PIA controllers were examined compared to conventional PI controllers. It has been shown that the designed PIA controllers can improve set-point control performance and disturbance rejection performance together. In this study, it has been observed that the RDR index, which is expressed by the energy spectral density of the controller function, can be effectively applied to improve the disturbance rejection performance of control systems. Thus, it can be said that the proposed PIA controller structure and the optimization method applied in control design applications will contribute to the increase of the robust control performance of the systems.

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