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Analytical and Dynamic study of Pulled Mass Nonlinear Vibration by Two Cables using Newton's Harmonic Balance Method

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Abstract: In recent years, much research has been done on nonlinear vibrations, and analytical and numerical methods have been used to solve complex nonlinear equations. The behavior of nonlinear oscillating equations is discussed until the second order is approximated. Harmonic balance method, which itself has limitations in application. This method continues to be able to study a wider range of nonlinear differential equations. In general, nonlinear vibration problems are of great importance in physics, mechanical structures, and other engineering research. First, the equation of nonlinear vibrations governing the mass of the particle mass connect to the drawn cable is calculated and then the Newton Harmonic Balance Method is used to study the nonlinear vibrations of the set and obtain the answer and its frequency. The method (NHBM) is done with Maple software and a comparison between the results of this method with the solution methods used by other researchers is shown to be a good match.

Keywords: Newton Harmonic Balance Method, Nonlinear Vibration, Frequency, Maple Software.

1. Introduction

Nonlinear vibration issues are of great importance in physics, mechanical structures and other engineering research. Vibration response, stability, and frequencies are the main components of a system's vibration check Therefore, investigating the influence of different parameters in these areas can be an important step in the design process. In the 1980s, many researchers used numerical or approximate methods to conduct research. It should be noted that the methods used by these researchers could not have predicted some important nonlinear phenomena such as subharmonic responses and turbulence. At certain intervals of vibration, the system has turbulent vibrations. In this case, the vibrations will be intense and unpredictable. One of the most common of these methods is the perturbation method, which involves expanding the series around a small parameter in a nonlinear system. Newton's harmonic methods are approximate, similar to the February series method used for linear oscillators. And it can be used to investigate the periodic solutions, which are as follows:

1) The solution showed a shortened February series.
2) Put the assumption solution in the equation of motion and expand each term into a series of February.
3) Higher frequency harmonics that are not in the original assumed solution.
4) Equilibrium coefficients for each February term (harmonic) This to a set of algebraic equations.
5) Solves algebraic equations.

Harmonic balancing methods are another common method in this field that have their own limitations in application. Due to these limitations in the use of methods, there is always an attempt to develop analytical and semi-analytical methods for the ability to study a wider range of nonlinear differential equations. Therefore, in this paper, Newton's harmonic balance (NHBM), which is a combination of Newton's method and harmonic balance method, is used to solve the problem of nonlinear vibrations of the mass of the particle connected to the drawn cable [1]. The subject of particle mass bound to non-mass cables has also been investigated by researchers using various solutions. One of the most recent studies was that of a senior researcher and Bele´ndez et al [2] who provided an approximate solution using the hypertrophy method of perturbation for the problem. Sehreh et al [3] also used an approximate solution using the maximum minimization method, the frequency amplitude frequency method, and the parameter expansion method to solve this problem. Akbarzadeh et al [4] also used non-linear vibration frequency amplitude frequency formula to investigate the mass connected to the drawn cables and compared the frequencies in different vibration ranges with the harmonic balance method. Khaled et al. [5] also used an approximate homeopathy method to solve the problem of nonlinear vibration of a mass connected to a drawn cable. Betaineh [6] used Homotopic Analysis Method (HAM) to solve the problem and find the periodic answer. This article also uses the NHBM method to facilitate the computational process.

2. Statement of the Problem

Figure 1 shows an L-shaped horizontal elastic beam and an E-shaped elastic module of level I, which are subjected to axial load P, uniform cross-sectional area, and assumed homogenous material. The beams are modeled according to Bernoulli's Euler beam theory. In the Bernoulli beam hypothesis, the cross-sectional plates remain flat after deformation, the midline will remain perpendicular to the middle plate. In this section, we obtain the differential equation of the transverse vibrations of the beam, using the energy method, and in Figure 1, the element shows the initial length of the dx from the beam after deformation and displacement.

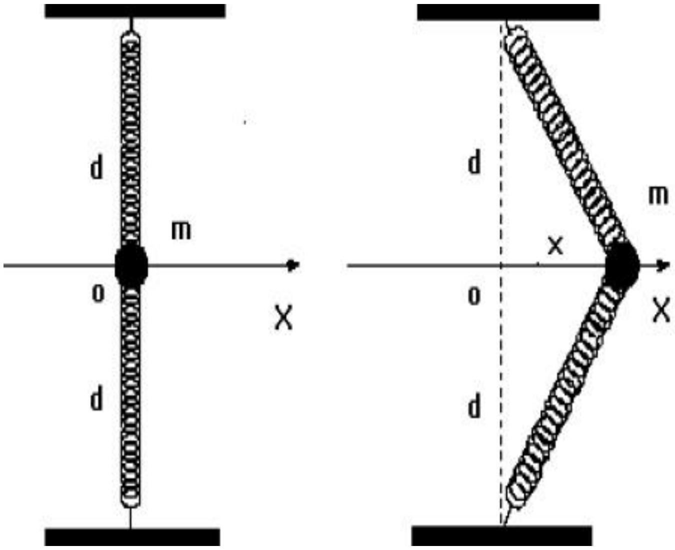


Fig.1. Mass connected to drawn cables [4].

The geometry of the problem is as shown in Figure 1. Consider the motion of a particle of mass m in the direction of x, which is connected to the cables drawn on both sides, and the cables are fixed at the end. If we apply Hooke's law to each piece of cable drawn, the T-tension in each part is equal to:

$$T=k(L-a) \qquad (1)$$

In this case, the length L is long, and the length a is x = 0 and k is the hardness factor. The sum of the forces acting in the direction of x on the mass m are as follows:

$$m\frac{d^2x}{dt^2} = -(2S)sin\theta = 2k(L-a)(\frac{x}{L}) \qquad (2)$$

Where sinϴ=(x/L). Since $L^2 = d^2 + x^2$ ، equation 2 can be written as follows:

$$m\frac{d^2x}{dt^2} + 2kx - \frac{2kax}{\sqrt{d^2+x^2}} = 0 \tag{3}$$

If x<<d means the amplitude of the vibrations is low, the relation 3 becomes simpler [7]:

$$m\frac{d^2x}{dt^2} + \left[\frac{2k(d-a)}{d}\right]x + \left(\frac{ka}{d^3}\right)x^3 + \cdots = 0 \tag{4}$$

The equation of motion is as follows:

$$\ddot{x}_{(t)} + \alpha x_{(t)} + \beta x^3_{(t)} = 0 \tag{5}$$

Where α and β are defined as follows:

$$\alpha = \left[\frac{2k(d-a)}{d.m}\right] \quad ، \quad \beta = \left(\frac{ka}{d^3.m}\right) \tag{6}$$

Equation 5 is the equation governing the vibrations of the set in Figure 1 and the set is in the following initial conditions:

$$x(0) = A \ \ . \ \ \dot{x}\,(0) = 0 \tag{7}$$

In this case, a determines the maximum amplitude of the cross-sectional vibration of the beam, which is shown in Figure 2 of Newton's harmonic balance method.

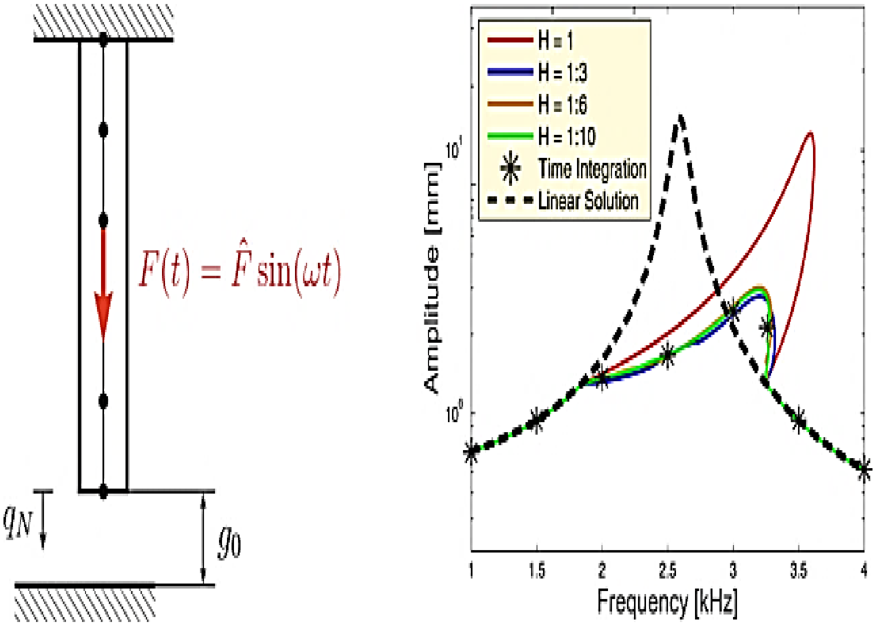
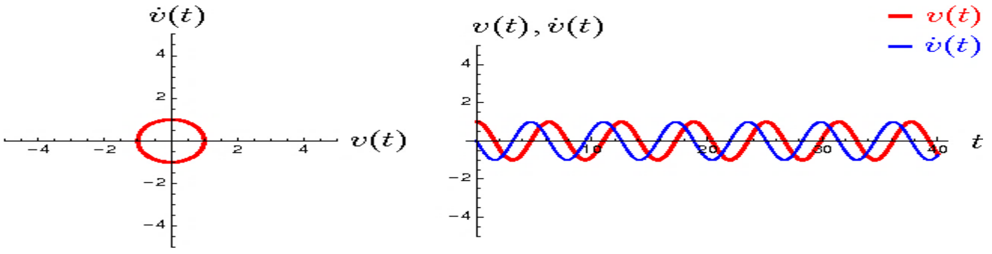


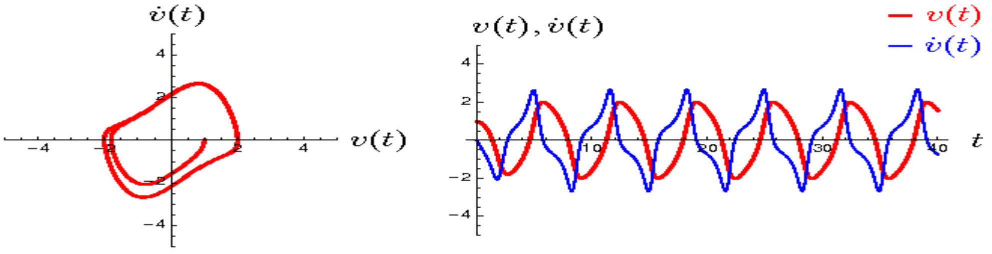
Fig. 2. Newton Harmonic Balance Method [6].

3. Newton Harmonic Balance Method

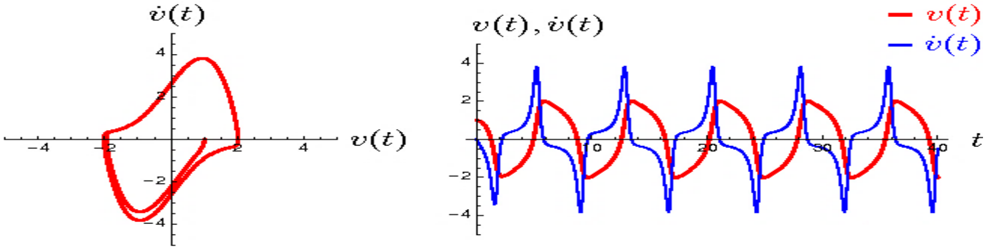
The Newton Harmonic Balance Method is a combination of the Newton method and the Harmonic Balance Method, first introduced in 2006 by Wu 2006 and shows figure 3 ways the Van der Pol oscillator method [8].



(a) $\epsilon = 0$ (linear)



(b) $\epsilon = 1$ (somewhat nonlinear)



(c) $\epsilon = 2$ (strongly nonlinear)

Fig. 3. Van der Pol oscillator method [6].

The equation of motion of a vibrational system is defined as follows:

$$\begin{aligned} &\frac{d^2x}{dt^2} + f\left(u.\frac{du}{dt}.\frac{d^2u}{dt^2}\right) = 0 \\ &u(0) = A \ \ . \ \ \frac{du}{dt}(0) = 0 \end{aligned} \tag{8}$$

Assume that f is a function of an individual in the equation, that is:

$$f\left(-u.-\frac{du}{dt}.-\frac{d^2u}{dt^2}\right) = -f\left(u.\frac{du}{dt}.\frac{d^2u}{dt^2}\right)$$

By placing τ = ωt in equations 3 and 4, rewrite them as follows:

$$\begin{aligned} &\omega^2 u'' + f\left(u.\omega u'.\omega^2 u''\right) = 0 \\ &u(0) = A\,.\,u'(0) = 0 \end{aligned} \tag{9}$$

In these relations, the prime shows the derivative of τ. Using Newton's method, the displacement and frequency squares are expressed by the relations, in which the relations Δu_1 and $\Delta\omega_1^2$ show a small increase in the principal values of displacement u_1 and the square frequency ω_1^2, respectively] [9] .

$$u(\tau) = u_1(\tau) + \Delta u_1(\tau) \tag{10}$$

$$\omega^2 = \omega_1^2 + \Delta\omega_1^2 \tag{11}$$

By placing relations 10 and 11 in Equation 12 for analytical approximation, the first order is as follows:

$$u_1(\tau) = Acos\tau \quad . \quad \Delta u_1 = \Delta u'' = \Delta\omega_1^2 = 0 \tag{12}$$

For our second approximation, we use Equation 13 After placing Equations 10 and 11 in Equation 9, we get the coefficients $cos3\tau$ and $cos\tau$ by zero and solve the equations of terms C_1 *and* $\Delta\omega_1^2$ simultaneously.

$$\Delta u_1 = C(cos\tau - cos3\tau) \tag{13}$$

Therefore, the approximation of the second order for frequency and displacement is obtained as follows [10]:

$$\omega = \sqrt{\omega_1^2 + \Delta\omega_1^2} \tag{14}$$

$$u(t) = (A + C)cos\omega t - Ccos3\omega t \tag{15}$$

4. Problem Solving using Newton's Harmonic Balance Method

Using the harmonic balance method to obtain the equation frequency, assume that the answer is z = Acos ωt, by placing the assumption answer in equation 5 to the following equation:

$$-A\omega^2 cos\omega t + A\alpha cos\omega t + \beta A^3\left(\frac{1}{4}cos3\omega t + \frac{4}{3}cos\omega t\right) = 0 \tag{16}$$

To obtain the frequency, $cos\omega t$ coefficients are equated on both sides:

$$\omega = \sqrt{0.75A^2\beta + \alpha} \tag{17}$$

Therefore, the first approximation of the equation was obtained in the form of $z1 = Acos(0.75A^2\beta + \alpha)^{1/2}t$, to obtain the second approximation of Newton's harmonic equilibrium method as stated by placing τ=ωt in the equation 16 is as follows.

$$\begin{gathered} \omega^2 z''_\tau + \alpha z_\tau + \beta z_\tau^3 = 0 \\ z(0) = A \ . z'(0) = 0 \end{gathered} \tag{18}$$

By placing relations 17 and 18 in Equation 19, the equation is written as follows:

$$(\omega_1^2 + \Delta\omega_1^2)(z_1'' + \Delta z_1'') + \alpha(z_1 + \Delta z_1) + \beta(z_1 + \Delta z_1)^3 = 0 \tag{19}$$

The following equation obtained by linear this equation with respect to Δu_1 and $\Delta\omega_1^2$:

$$z_1''(\omega_1^2 + \Delta\omega_1^2) + \Delta z_1''\omega_1^2 + \alpha(z_1 + \Delta z_1) + \beta(z_1^3 + 3z_1^2\Delta z_1) = 0 \tag{20}$$

To obtain the approximation of the second order, place the answer of Equation 20 in place of Δz_1 in Equation 21, then obtain the following equations using the coefficients of $cos\tau$ and $cos3\tau$:

$$-(A + 2C)\omega_1^2 - A\Delta\omega_1^2 + \alpha(A + C) + \frac{4}{3}\beta(A^3 + 3CA^2) = 0 \tag{21}$$

$$C\omega_1^2 - \alpha C + \frac{1}{4}\beta(A^3 + 3CA^2) = 0 \tag{22}$$

By solving this equation simultaneously for equations $\Delta\omega^2$ and C, the following relations reached:

$$C = \frac{-\beta A^3}{4\omega_1^2 - 4\alpha + \frac{16}{3}\beta A^3} \tag{23}$$

$$\Delta\omega_1^2 = \frac{\alpha}{A}(A + C) + \frac{4}{3}(\beta A^2 + 12C) - \frac{(A + C)\omega_1^2}{A} \tag{24}$$

Therefore, the approximation of the second order for frequency and displacement by Newton harmonic balance method will be as follows:

$$Z = (C + A)\cos\omega t - C\cos 3\omega t = A\cos\omega t + \frac{-\beta A^3}{4\omega_1^2 - 4\alpha + \frac{16}{3}A}(\cos\omega t - \cos 3\omega t \tag{25}$$

$$\omega = \sqrt{\omega_1^2 + \Delta\omega_1^2} = \sqrt{\omega_1^2 + \frac{\alpha}{A}(A + C) + \frac{4}{3}(\beta A^2 + 12C) - \frac{(A + C)\omega_1^2}{A}} \tag{26}$$

5. Results and Discussion

In this section, first, the accuracy of the obtained relations is compared with the researches and then by drawing the equations obtained by maple software, the effect of coefficients and parameters on the frequency and response is investigated. Table 1 compares the results of the HBM and He’s amplitude frequency formulation methods investigated in reference [4] with the method used in the present paper, which shows a good correlation in the results due to the small difference.

Table 1: Comparison of NHBM method frequency with reference [4].

HBM	**NHBM**	**A**
0.7071	0.7071	0.01
0.7075	0.7075	0.05
0.7085	0.7086	0.1
0.7126	0.7124	0.2
0.7190	0.7188	0.3
0.7369	0.7374	0.5
Haff	**Difference with the method HBM**	**Difference with the method Haff**
0.7071	0	0
0.7074	0	0.0001
0.7074	0.0001	0.0002
0.7124	0.0002	0
0.7189	0.0002	0.0001
0.7395	0.0005	0.0021

After verification, the effect of the coefficients on the system frequency is investigated using diagram, which shows the behavior of dimensionless deviation z(A,t) and $\alpha = \pi$ and $\beta = 0.15$ and the maximum displacement in the middle of the beam is equal to five. In Figure 4, the frequency changes with respect to the amplitude by changing β in $\alpha = 14.3$ are dimensionless and in Figure 5, the displacement obtained by the NHBM method in $\alpha = \pi$, $\beta = 0.15$ and in Figure 6, the frequency changes with respect to the amplitude by changing α in $\beta = 0.15$ are parameters, and in Figure 7, the frequency changes with respect to the $\alpha = \beta = 0.15$ amplitude are shown.

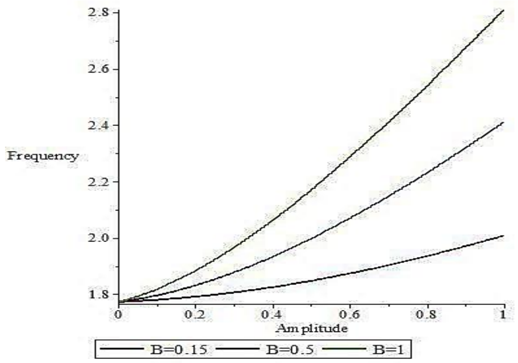


Fig. 4. Frequency changes to the amplitude by changing β at α = 14.3

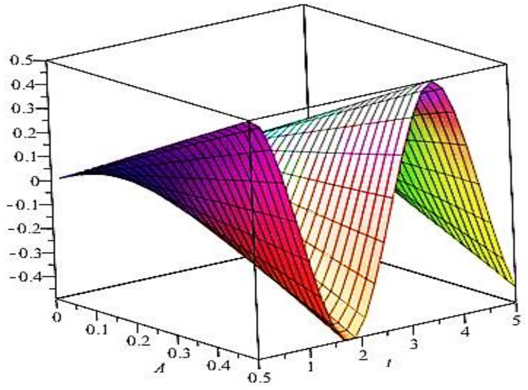


Fig. 5. Displacement obtained by NHBM method at α = π and β = 0.15

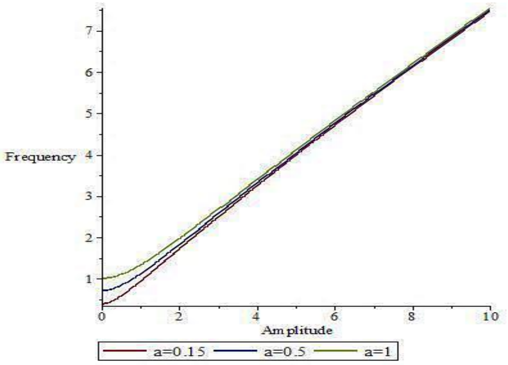


Fig. 6. Frequency changes with the amplitude change α at 0.15 = β

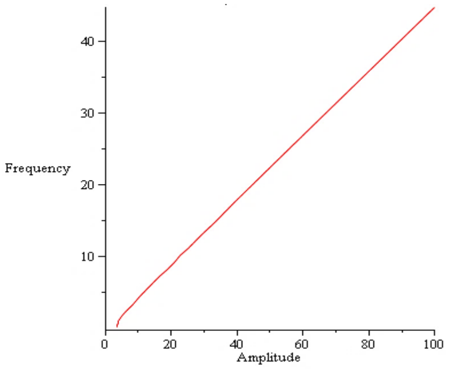


Fig.7. Frequency changes to the amplitude α = β = 0.15

6. Conclusion

This paper investigates the response to nonlinear vibrations of a mass connected to a cable. After obtaining the equation governing the vibrations of the beam, a new Newton harmonic balancing method (NHBM) is used to solve this equation and complex dynamic behavior, such as a drastic change in the output regime, predicts a sudden instability. Using the Newtonian harmonic equilibrium (NHBM) method for quantitative analysis, the frequency response is predictable and it was found that the frequency of nonlinear free vibration decreases with increasing initial tensile ratio and control parameters (pressure and voltage). Comparing the results of this method with the work of other researchers shows that this method has a good accuracy.

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