

Basic research for designing the erbium doped fiber amplifier in long distance cable communications

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Abstract. The paper presents some of the author results obtained in the research on the optical fiber amplifiers and Quantum Well (QW) laser diodes used in long distance optical communications as those transoceanic and other applications. Based on the mathematical models of the Erbium Doped Fiber Amplifier (EDFA) and of the QW laser diodes, implemented in software (especially, MATHLAB and MATHCAD but also in SIMULINK and SPICE), we get the operational diagrams and parameters of the specified devices and structures. For example, the classic Runge-Kutta method has been successfully used for numerical simulation of complex nonlinear dynamics in fiber optic lasers. The obtained results are important in the design and characterization of the optical fiber communication systems, as those of the optical cables used in the naval and civil engineering.

1. Introduction

The development of the advanced optical technologies for transmitting telephone signals, Internet communication and cable television in a multitude of applications with industrial, commercial, medical, and defense/governmental character for data storage as well as in naval engineering, laid to a vast network of optical fiber and cable communication lines, in long-distance applications, intercity and transoceanic [1]. An intercontinental network of 250,000 km of submarine communication cables for long-distance applications, intercity and transoceanic with a capacity of 2.56 Tb/s was completed, by 2002 [2, 20]. Modern lines of communications use the Erbium Doped Fiber Amplifier (EDFA) instead of the classical repeaters so that distances of more than 100 kilometers, are permitted between the amplification points [3-7]. These lines of communications use also the wavelength-division multiplexing of many signals so that the capacity of the fiber channels is increased. The data traffic that is crossing the oceans is currently carried 99% by undersea cables. The total carrying capacity of submarine cables is in the range of terabits per second, while satellites typically offer only 1,000 megabits per second, so that the superiority of the submarine cables in comparison with other systems of information transmission is demonstrated. The paper presents the author's research on optical fiber amplifiers and Quantum Well Lasers (QWL) diodes, the results being important in communications, especially those used in long distance transoceanic and intercity systems for information transmission [8, 27, 30].

The paper intend to start from mathematical models of the optical fiber amplifiers and QWL diodes implemented in software, especially, MATHLAB, MATHCAD and other programming tools to get the operational diagrams and parameters for the devices and structures used in the optical fiber communications systems [9, 12-16]. After introduction we shall analysis the basic models and structures of the erbium doped fiber amplifiers. The presentation includes the results of some

representative numerical simulations and their interpretation as well. The same problems for the quantum well laser diodes are analysed in the next part of paper, before the conclusions [28-30].

2. Erbium doped fiber amplifiers

2.1. Basic models and structures

Erbium-doped fiber optic amplifier systems (EDFAs) operate around the wavelength range in which losses in silica fibers are minimal. Because in such a system the power of incident light is amplified by stimulated emission, using the same mechanism as in the laser, an optical amplifier can be considered a "laser without feedback". Erbium doped optical fiber systems have important advantages for transmission and processing of information compared to the classical ones: the possibility of integration; high efficiency and profit; immunity to inter modulation; low noise, output high saturation power [1]. The main element of the amplifier is a piece of optical fiber doped with erbium ions, which is pumped by a high power laser diode to create the population inversion between the implied levels. The signal wave and the pump wave are both coupled to the optical fiber by means of a wavelength division multiplexer (WDM). Optical isolators are needed to reduce reflections, ie the return of photons in the amplification environment. The realization of such an amplifier requires the selection of optical components (doped fiber, pumping laser, isolators, multiplexing device in wavelengths), optimizing and assembling them so as to result in minimal losses and reflections in the obtained system. The main analytical and numerical models we have used for the study of fiber optic amplifiers are [17-19]: the transport equations model for the analytical and numerical modeling of the EDFA amplification regime and the computational model, used for the analytical and numerical modeling of interaction phenomena by varying the parameters of the fiber optic laser.

The model of the transport equations is represented by the rate equation for the population of the higher laser level to which the signal and pump equations are added. Through the numerical integration of the rate equations for the signal and pump intensities, we aimed, mainly, to explain the mechanism of limiting the rate of increase of the amplification during the propagation of the useful signal through optical fiber doped with erbium.

It was found a limitation of the rate of increase of the amplification during the propagation of the useful signal by optical fiber doped with erbium, which was explained by the decrease of the population $N_2(z)$ of the upper laser level with the decrease of the pumping intensity $I_p(z)$ with the passage through the active region of the amplifier (doped with erbium), a decrease noted in the curves drawn by numerical integration. The obtained results can be used in the design, for example, to determine the optimum amplifier length for maximum efficiency.

The computational model. The model used for the system studied is determined by the energy diagram with 7 levels of the erbium, in different host environments so it includes nine differential equations, which can be solved by numerical methods. where N_i ($i = 0, 1, 2, 3, \dots, 6$) are the population densities of levels and ϕ designate the photon densities.

The study of the influence of the host material on the laser output power required a three-dimensional study, each host material being characterized by three parameters: the life times of the levels, the "up-conversion" parameters and the effective pumping section.

As a result, the variations of these parameters must be correlated, taking into account the relative weights of each, which is possible only through judicious graphic representations

In figure 1 are shown energy levels of erbium in the host material and the corresponding transitions between them:

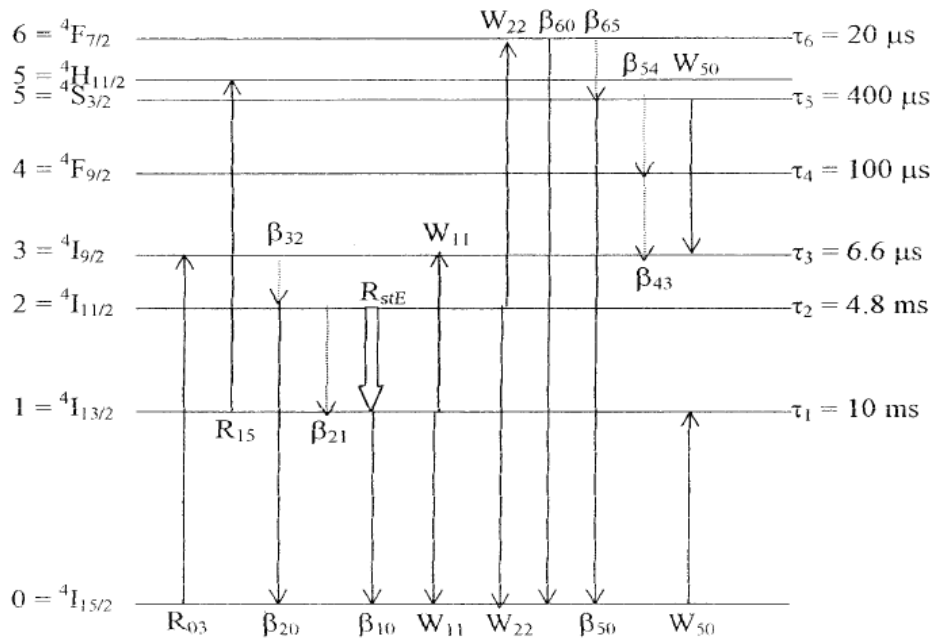


Figure 1. Diagram of energy levels of the Er^{3+} [18].

The main phenomena studied by computer experiments are: the amplification of the modulated optical signal in EDFA; laser efficiency and threshold for different pumping wavelengths; the dependence of the power output on the life time of the levels; the influence of host materials on output power and other design parameters; stability and non-chaotic operating regime; the nonlinear dynamics in optoelectronic systems for coherent radiation amplification. Some of the obtained results will be presented in the section 2.3.

The model of coupled nonlinear equations[30]:

$$\frac{dI_p}{dz} = -\sigma_p^a \cdot I_p N_0 \cdot \frac{\frac{\sigma_p^a \cdot I_p}{h\nu_p} + \frac{\sigma_s^a \cdot I_s}{h\nu_s}}{\frac{\sigma_p^a \cdot I_p}{h\nu_p} + \frac{\sigma_s^a \cdot I_s}{h\nu_s} + \frac{1}{\tau} + \frac{\sigma_s^e \cdot I_s}{h\nu_s}} \quad (2.1)$$

$$\frac{dI_s}{dz} = \sigma_s^a \cdot I_s N_0 \cdot \left[\frac{\sigma_s^e + \sigma_s^a}{\sigma_s^a} \cdot \frac{\frac{\sigma_p^a \cdot I_p}{h\nu_p} + \frac{\sigma_s^a \cdot I_s}{h\nu_s}}{\frac{\sigma_p^a \cdot I_p}{h\nu_p} + \frac{\sigma_s^a \cdot I_s}{h\nu_s} + \frac{1}{\tau} + \frac{\sigma_s^e \cdot I_s}{h\nu_s}} - 1 \right] \quad (2.2)$$

was used to study the amplification of the input modulated optical signal.

The performances of such an amplifier were studied by numerically solving the system of equations (2.1) and (2.2) as is below.

2.2. Numerical results and discussions

2.2.1. The amplification . Numerical modeling of the upper rate equations was realized using the MATLAB programming medium. The basic element of the program was the function *ode 45*, which realize the integration of the right side expressions of the nonlinear coupled equations using Runge - Kutta type methods, for calculation time reducing. The program was applied for many values of the

amplifier length for each of them resulting different sets of results, for the photon fluxes, both for the signal and pumping as well as for the gain coefficients and signal to noise ratio. From the obtained results by numerical integration of the transport equations, it results that the intensity of the output signal rise with the amplifier length but the pumping diminish in the some time. The calculated gain coefficients of the amplifier have a similar variation as was expected. We observe also the rising of the signal to noise ratio, resulting an improving of the amplifier performances [17, 26]. The obtained value of the gain coefficient for the signal, of the 40 dB is similar to published values [1], so that, results can be very useful for designers, for example, to calculate the optimum length of the amplifier for maximum efficiency.

2.2.2. *The influence of the Er^{3+} ion doped host material on the output power.* The analyzed physical system was the optical fiber with ZBLAN composition, having the next characteristic parameters [18] - the dopant concentration: $N_d : 1,8 \cdot 10^{19} \text{ cm}^{-3}$; the amplifier length, $l : 480 \text{ cm}$; the laser mod radius, $r_{\text{mode}} : 3,25 \mu\text{m}$; the pumping wavelength, $\lambda_p : 791 \text{ nm}$; the ground state absorption cross-section, $\sigma_{03} : 4,7 \cdot 10^{-22} \text{ cm}^2$; the excited state absorption cross-section from the level $^4I_{13/2}$, $\sigma_{15} : 10^{-21} \text{ cm}^2$; the excited state absorption cross-section from the level $^4I_{11/2}$, $\sigma_{27} : 2 \cdot 10^{-22} \text{ cm}^2$; the laser wavelength, $\lambda_L : 2,71 \mu\text{m}$; the "colaser" wavelength, $\lambda_{cl} : 1,7 \mu\text{m}$; the emission cross-section, $\sigma_{21} : 5,7 \cdot 10^{-21} \text{ cm}^2$; the "colaser" cross section, $\sigma_{53} : 0,5 \text{ or } 0,1 \cdot 10^{-20} \text{ cm}^2$; the Boltzmann, b_{14} and $b_{22} : 0,113$ respectively $0,2$; the mirror transmission $T : 68\%$; the optical resonator length, $l_{\text{opt}} : 720 \text{ cm}$.

The "colaser" process was studied for three different values of the "colaser" cross - section: $\sigma_{53} = 0 \text{ cm}^2$; $\sigma_{53} = 0,5 \cdot 10^{-20} \text{ cm}^2$ and $\sigma_{53} = 0,1 \cdot 10^{-20} \text{ cm}^2$.

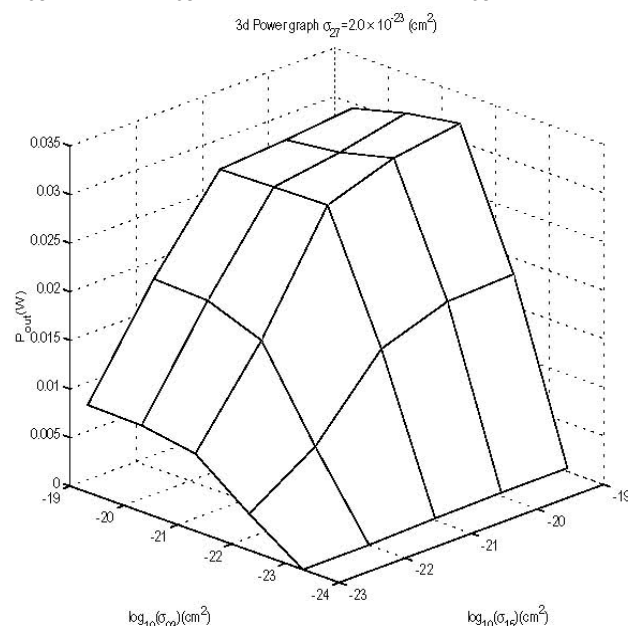


Figure 2. Dependence of the output laser power both on σ_{15} and σ_{03} cross sections for a value of σ_{27} [18].

This figure a strong dependence of the laser power on σ_{15} and σ_{03} cross sections in terms of the high life time of the lower laser level τ_1 at a sufficiently low value of the effective cross section,

$$\sigma_{27} = 2 \cdot 10^{-23} \text{ cm}^2.$$

2.2.3. *Numerical results regarding nonlinear effects in fiber optic systems.* The self - pulsation and the stability of the functioning were studied in the Erbium doped systems "crystal type" or "fiber optic type" in the approximation of the rate equations [8, 30].

We used numerical models of the modified rate equations to consider up-conversion phenomena as a result of ion pair formation, at high dopant concentrations [10,11].

The model of ion pairs in the fiber laser is useful to explain the nonlinear dynamics of this type of laser observed experimentally, which refers to the increase of the laser threshold, the decrease of the gain and the operation in self-pulsed regime. To limit the undesirable effects of nonlinear dynamics, low dopant concentrations should be used so that the mean distance between ions is large enough to impede their interaction, with practical consequences of increasing fiber length for the same amplification performances.

The results are in accordance with the existing data in the literature, obtained theoretically or experimentally. Long term temporal evolution of laser intensity at the fraction $x = 0.1$ of ion pairs and two levels of the pumping level is shown in figure 3.

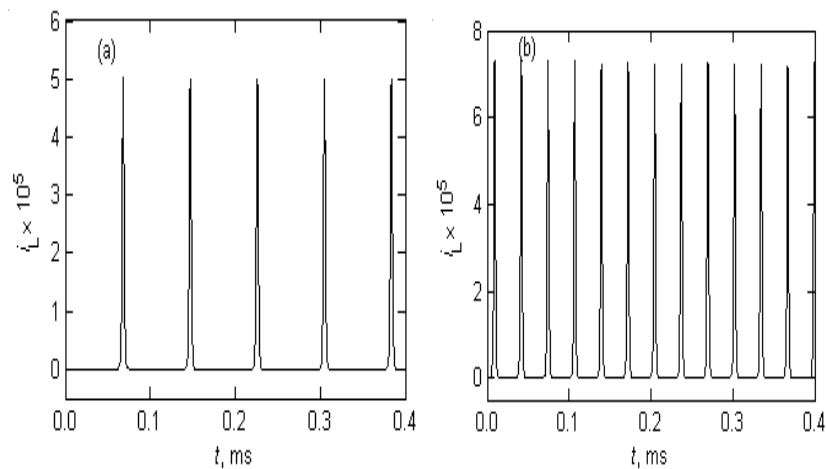


Figure 3. Long term temporal evolution of laser intensity at the fraction $x = 0.1$ of ion pairs and the pumping level (a) $r = 1.5$, (b) $r = 2.5$, $\tau_L = 10^{-8}$ s. [18].

3. Quantum Well Laser Diodes

3.1. Basic models and structures

The use of lasers with quantum wells allows the control of the parameters that govern their operating characteristics and offers flexibility in design, widening the possibilities of engineering in this field of nanotechnologies. [21-25].

In the approximation of the rate equations, the dynamics of the laser is described using the injected electron density N and the density of photons S in the form [25]:

$$\frac{dN}{dt} = \frac{I}{qV_{act.}} - g_0(N - N_0)S - \frac{N}{\tau_n} \quad (3.1)$$

$$\frac{dS}{dt} = g_0(N - N_0)S - \frac{S}{\tau_p} + \gamma \frac{N}{\tau_n} \quad (3.2)$$

The terms involved in the above equations have the meanings: the injection rate of the carriers; the rate of decrease of the carriers due to the stimulated emission; the rate of decrease of the carriers due to the spontaneous emission; photon growth rate through stimulated emission; photon decrease rate due to cavity losses; spontaneous emission rate inside the laser mode.

A comparative study of the models presented for lasers with quantum holes is possible by numerically integrating the equations corresponding to different types of pumping signals in MathCad. for both linear and nonlinear models.

The model with nonlinear saturation of gain was integrated for different injection currents. The corresponding waveforms for the optical output power were obtained for: constant, sinusoidal and rectangular injection current.

In general, all the studied models give satisfactory results for the study of the transient and dynamic regime at low injection levels. At high injection levels, the numerical results obtained highlight the specific limits of each model, being in accordance with the theoretical analysis. SIMULINK simulation allows to find the answer for different waveforms of the signal used as input. The results were published [16].

The quantum well laser modeling have been used for numerical simulations of quantum well laser features using the MATHCAD program and the Runge - Kutta integration algorithm for the study of: dynamic regime and frequency characteristics; the response to different types of signals; small signal model; frequency response; laser diode modulation; nonlinear dynamics of laser with quantum wells; transfer function; chaos routes and bifurcation diagrams.

3.2. Numerical results and discussions

The study of the functional characteristics of QWL is possible based on the model presented above and others [27], using MATHCAD programs for integrating these equations, using the appropriate algorithms. A comparative study of the models presented for lasers with quantum wells is possible by numerically integrating the corresponding equations for different types of pumping signals [28-30].

In general, all the models mentioned lead to satisfactory results for the study of the transient and dynamic regime at low injection levels. At high levels of injection, the numerical results obtained highlight the specific limitations of each model, being in accordance with the theoretical ones.

3.2.1. Frequency response. A small signal model was obtained in order to be able to analyze the frequency response of a QW laser for different polarization currents. The system of linear equations of the small signal model written in a matrix form easily implemented in MATHCAD has the form (3.3). The values of the model parameters are given in [27].

$$\begin{pmatrix} \frac{-i\Delta I}{qV_{act}} + i(1-\varepsilon S)Sg_0(N-N_0)\Delta P \\ -i(1-\varepsilon S)Sg_0\Gamma(N-N_0)\Delta P \end{pmatrix} = \begin{pmatrix} \bar{\omega} - ig_0S(1-\varepsilon S) - \frac{i}{\tau_n} & -i(1-2\varepsilon S)g_0(N-N_0) \\ i\Gamma g_0S(1-\varepsilon S) + i\Gamma\beta\frac{1}{\tau_n} & \bar{\omega} + i(1-2\varepsilon S)g_0\Gamma(N-N_0) - \frac{i}{\tau_p} \end{pmatrix} \cdot \begin{pmatrix} N \\ S \end{pmatrix} \quad (3.3)$$

3.3.2. Chaos routes and bifurcation diagrams. Knowing the hierarchy of instabilities of the laser devices is useful in designing and calculating of the operating stable regimes in applications. In figure 4 is presented the bifurcation diagram of the density of photons versus the modulation index as an indicator of the system evolution.

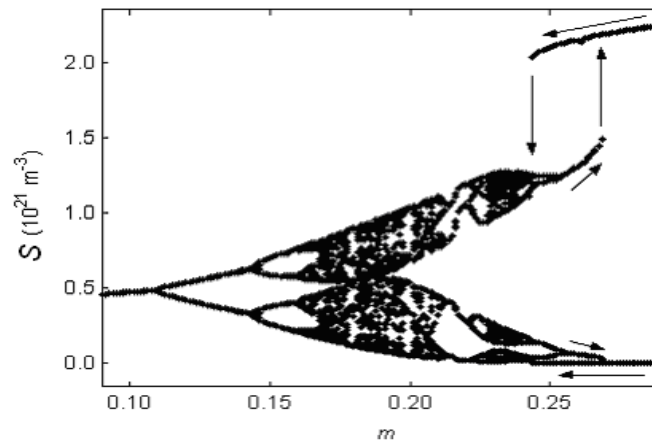


Figure 4. The diagram of bifurcations for the photon density versus the modulation index [27].

In the chaotic region, the existence of periodic dynamic windows and a hysteresis type evolution (marked with arrows on the figure) are noted. The superior performance of lasers with quantum wells compared to ordinary laser diodes, recommends this type of device for high-speed in optical communications, provided that undesirable effects such as bifurcation points and chaos are avoided.

4. Conclusions

Based on the mathematical models of the EDFA type optical amplifier and of the QW laser diodes, implemented in software (especially, MATHLAB and MATHCAD but also SIMULINK and SPICE), we get the operational diagrams and parameters of the specified devices and structures.

The studies carried out aimed to develop a computational method to confirm the existing data in the specialized, theoretical, experimental and numerical literature. Another objective of the researches was the problem of discovering new situations in which, the laser operating conditions can be more easily fulfilled, taking into account the valences of the "experiment computer" methods used or to improve the operating performance of such devices by design, based on new correlations between constructive and functional parameters.

The nonlinear dynamics of an erbium-doped fiber laser was explained based on a simple model of the ion pairs present in heavily doped fibers. Depending on the ion pair concentration, the pumping level and the photon lifetime in the laser cavity, numerical calculations predicts a sinusoidal dynamics or a self-pulsing one. To limit the undesirable effects of nonlinear dynamics, low dopant concentrations should be used so that the mean distance between ions is large enough to impede their interaction, with practical consequences of increasing fiber length for the same amplification performances. The results obtained are in accordance with the existing data in the literature, obtained theoretically or experimentally.

The numerical calculations we have performed also referred to nonlinear effects in laser systems with quantum wells in the presence of modulation. In the case of increasing the modulation index and by approaching the modulation frequency to the relaxation oscillations frequency, a pulse operating regime is obtained, with a duration of the order of tens of picoseconds (ultra-short pulses), the laser being used as a source of pulses for optical communications.

For quantum well lasers we have highlighted their complex nonlinear dynamics, characterized by bifurcation points and chaos, the critical values of the parameters being determined. Computing software has been developed to study nonlinear phenomena in this kind of devices. The MATHCAD program has proven to be a powerful tool in analyzing and designing systems according to these

models. The classic Runge-Kutta method has been successfully used for numerical simulation of complex nonlinear dynamics in fiber optic lasers.

Laser modulation studies are important also for the generation of solitons in optical fibers, the phenomenon of phase auto-modulation being the essential mechanism in this process. The study of the interaction between two solitons using a powerful computer program (Maple10) [7, 28] is of great interest for a better understanding of the propagation properties of solitons in optical fibers used in high speed data transmissions to reduce dispersion effects.

The nonlinear phenomena studied are of interest also for the developing research in Romania regarding the project "Extreme Light Nuclear Physical Infrastructure" (ELI-NP) on the realization of the high power laser systems of petawatt and ultrashort pulses, together with the gamma collimated system, where due to the high energies involved, nonlinear phenomena are important and cannot be avoided in designing and functioning.

The obtained results are important in the design and characterization of the modern optical communication ensembles, as the optical cables used in the naval and civil engineering.

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