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Computing the Cost of Service Projects in Telkaif Town Using Soft Sets and Soft Topology

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Abstract. In this research, we will present a new method of collecting data and performing calculations on it to obtain the required results differently from the usual, at the beginning we will define new concepts of soft sets in soft topological spaces and we will give and explaining many properties of them as soft ii- dense in itself sets, soft ii-connectedness, soft ii-perfection and soft ii-compactness. After that, in the practical part, we will use the concept of soft topology in calculating the cost of service projects in several areas of Telkaif district of Nineveh Governorate in the north of Iraq, whether the cost of projects is in the one area or several areas or all of them using the theory of soft sets in soft topological spaces.

Keywords. Soft sets, Soft topology, Cost of service projects, Telkaif town.

Introduction

Soft topology is a relatively new and essential subject of study that has the potential to transform the way we think about the world by enhancing creative and inventive ways as well as mathematical models that really are revolutionary aid in the solving of complex topics in a range of fields. Molodtsov proposed the notion of soft set in 1999 as follows: if X is an initial universe.' and $P(X)$ signifies X's power set, E is a set of parameters and ($\emptyset \neq A \subseteq E$). Over X a soft set is known as a couple (Q,P), where Q is regarded as mapping $Q: A \to P(X)$. A parameterized collection with X subdivisions is known as a soft set. In Specific $p \in A$, $Q(p)$ considered soft set, a collection of approximate factors (Q,A), whether $p \notin A$, so $Q(p) = \emptyset$, i.e. $Q_A = \{Q(p): p \in A \subseteq P, Q: A \to P(X)\}$. $SS(X_A)$ is the collective name for all of these soft sets over X (see[1]). After that, Molodtsov collaborated with a number of other academics to apply the idea theory to a variety of domains and directions (see [2], [3]).

Many studies have examined over an initial universe into the concept of soft topological spaces (see [4], [5], [6] and [7]).

Mohammed and Askandar proposed the idea of i-open sets in bi-topological spaces in 2018(see [8]). In 2020, 2022, Askandar S.W. and Mohammed A.A. introduce the notions of soft i-open, soft ii-open sets and soft ii-mappings in topologically soft spaces (see [9], [10]).

We know that data mining is the process of examining a vast amount of data to identify a logical relationship that presents data in a new way that is understandable and valuable to the data owner.

Mahmood, M.H., used the soft set and soft topology to assess the cost of infrastructure improvements in numerous residential zones in the governorate of Baghdad city in 2015, and found a lot of data (see [11]).

In this work, at the beginning in the theoretical part, we introduced a new kind of soft sets in soft topological spaces with some of its properties by proofs. In the practical part we will use the idea of a soft set to compute the Cost of *k* Service Projects in *n* regions in any city in general. After that we will apply this method specially to compute the cost of four service projects in five regions in Telkaif the small town in the north of Iraq.

(X, τ,E) is used to represent soft topological space throughout this study (in short *sTs)*.Soft open sets and soft closed sets denotes by *s-Os, s-Cs* respectively.

1. Theoretical Side of Soft Sets: Soft ii-Perfect set, Soft ii-Connectedness, Soft ii-Compactness.

Definition1.1. [9], [10] Let (Q,E) to be a soft set in(X,$\boldsymbol{\tau}$,E), therefore, (S,E) is said to be Soft ii-open set (*sII-Os*) if there is a soft open set $(\boldsymbol{J},\boldsymbol{E}) \neq \emptyset, \boldsymbol{X}$ where $(S,E)\tilde{\subseteq}Cl((S,E)\ \tilde{\cap}\ (J,E))$ and $Int(S,E)=(J,E)$.

The complement of soft ii-open set known as soft ii-closed set **(***sII-Cs***)**. The intersection of all soft ii-closed sets over *X* including *(S,E)* is the soft ii-closure of *(S,E)*, and it is represented by the symbol *sII-Cl(S,E).* A soft ii-interior of a soft set *(S,E)* is the union of all whole soft ii-open sets over *X* included in *(S,E),* and it is denoted by *sII-Int(S,E)*. The *sII-Os*, *sII-Cs* symbol denotes the group of complete soft ii-open sets and soft ii-closed sets in *(X, τ, E)*.

Remarks1.2. Let$(\boldsymbol{S},\boldsymbol{E})\ \tilde{\in}\ \boldsymbol{SS}(\boldsymbol{X_E})$and $m \in X$, consider$\emptyset \neq \boldsymbol{G} \subseteq \boldsymbol{X}$. We define:

1. $(S, E)\ \tilde{\setminus}\{x\}=\{S(r)\setminus\{m\}: \forall \boldsymbol{r} \in \boldsymbol{E}\ \}$.

2. $(S, E)\tilde{\cup}\ \boldsymbol{G} = \{\boldsymbol{S}(\boldsymbol{r}) \cup \boldsymbol{G}: \forall \boldsymbol{r} \in \boldsymbol{E}\}$.

3. $(S, E)\tilde{=}G$ if and only if $S(r)=G$, $\forall \boldsymbol{r} \in \boldsymbol{E}$.

4. $(S, E)\tilde{\cup}\ \{\boldsymbol{m}\} = \{\boldsymbol{S}(\boldsymbol{r}) \cup \{\boldsymbol{m}\}: \forall \boldsymbol{r} \in \boldsymbol{E}\}$.

5. $(S, E)\tilde{\cap}\ \{\boldsymbol{m}\} = \{\boldsymbol{S}(\boldsymbol{r}) \cap \{\boldsymbol{m}\}: \forall \boldsymbol{r} \in \boldsymbol{E}\}$.([9]), ([10]), ([12]), ([3]).

Definition1.3. [9], [10] Let$(S, E)\ \tilde{\in}\ SS(X_E)$. A point m ∈ X is an ii-limit point of *(S,E)* whether each *sII-Os (S,E)* which include x, the following is $(S,E)\ \tilde{\cap}\ (G, E)\tilde{\setminus}\{m\} \neq \emptyset_E$. ii-derived set of *(S,E)* designated by *II-D(S,E)is t*he collection of all ii-limit points of *(S,E).*

Definition1.4. A soft set (S,E) in (X,$\boldsymbol{\tau}$, E) is said to be:

1. 9], [10] Soft ii-closed set *(sII-Cs*) if:

a) $II\text{-}D(S,E)\ \tilde{\subseteq}(S,E)$ where $II\text{-}D(S,E)\subseteq S(r)$, $\forall \boldsymbol{r} \in \boldsymbol{E}$. b) *sii-Cl(S,E)=(S,E).*

2. Soft ii-dense in itself if $(S,E)\ \tilde{\subseteq}\ ii\text{-}D(S,E)$ where $S(r) \subseteq II\text{-}D(S,E)$, $\forall \boldsymbol{r} \in \boldsymbol{E}$.

3. Soft ii-perfect set if (*S,E*)= *ii-D(S,E)* where *S(r)=II-D(S,E)*, $\forall \boldsymbol{r} \in \boldsymbol{E}$. In other words, (*S,E*) is said to be Soft ii-perfect set if it is Soft ii-closed and Soft ii-dense in itself set.

Example1.5. Consider *X={q,t,z}*, $\tau = \{\phi_E, (S_1,E), (S_2,E), (S_3,E), X_E\}$, *E={r,s}*.Where$(S_1,E) = \{(r,\{q\}), (s,\{q\}),$
$(S_2,E) = \{(r,\{t\}), (s,\{t\})\}$,
$(S_3,E) = \{(r,\{q,t\}), (s,\{q,t\})\}$.
s-Os(X_E)= $\{\phi_E, (S_1,E), (S_2,E), (S_3,E), X\}$,
sCs(X_E)= $\{X, (S_1,E)^c = (S_4,E) = \{(r,\{t,z\}), (s,\{t,z\})\}, (S_2,E)^c = (S_5,E) = \{(r,\{q,z\}), (s,\{q,z\})\}, (S_3,E)^c = (S_6,E) = \{(r,\{z\}), (s,\{z\})\}, \phi_E\}$.
$\{\phi_E, (S_1,E), (S_2,E), (S_3,E), (S_4,E), (S_5,E), X\} \tilde{\in} sI - Os(X_E)$, also are$sII - Os(X_E)$.
$\{\phi_E, (S_1,E), (S_2,E), (S_4,E), (S_5,E), (S_6,E), X\} \tilde{\in} sII - Cs(X_E)$
Consider$(S_L,E) = \{(r,\{q,z\}), (s,\{q,z\})\}$,$(S_K,E) = \{(r,\{q,t\}), (s,\{q,t\})\}$

1. *II-D*(S_L,E)=$\emptyset \tilde{\subseteq} (S_L,E)$. Obviously (S_L,E) considered as *sII-Cs*, but it is not dense in itself set. Hence, (S_L,E) is not soft ii-perfect set.
2. *II-D*(S_K,E)=$\{z\} \tilde{\nsubseteq} (S_K,E)$. Clearly (S_K,E) is not *sII-Cs*, and it is not dense in itself set. Hence, (S_K,E) is not soft ii-perfect set.

Definition1.6. If (*S,E*) is a soft set in (*X*,$\boldsymbol{\tau}$, *E*), then, (S_1,*E*) and (S_2,*E*) represent a soft ii-separation for (*S,E*) and it is designated by $(\boldsymbol{S},\boldsymbol{E}) = \langle(\boldsymbol{S_1},\boldsymbol{E})|(\boldsymbol{S_2},\boldsymbol{E})\rangle$if they satisfied the following conditions:

1.$(\boldsymbol{S_1},\boldsymbol{E})$, $(\boldsymbol{S_2},\boldsymbol{E}) \tilde{\neq} \emptyset_{\boldsymbol{E}}$.

2.$(\boldsymbol{S_1},\boldsymbol{E}) \tilde{\cap} (\boldsymbol{S_2},\boldsymbol{E}) = \emptyset_{\boldsymbol{E}}$,

3. $(\boldsymbol{S_1},\boldsymbol{E}) \tilde{\cup} (\boldsymbol{S_2},\boldsymbol{E}) = (\boldsymbol{S},\boldsymbol{E})$.

4. $(\boldsymbol{sII - Cl}(\boldsymbol{S_1},\boldsymbol{E}) \tilde{\cap} (\boldsymbol{S_2},\boldsymbol{E})) \tilde{\cup} ((\boldsymbol{S_1},\boldsymbol{E}) \tilde{\cap} \boldsymbol{sII - Cl}(\boldsymbol{S_2},\boldsymbol{E})) = \emptyset_{\boldsymbol{E}}$,

Definition1.7. If a soft set *(S,E)* in (X,τ,E)without a soft ii-separation is said to be a soft ii-connected. In other words, (*S,E*) is said to be a soft ii-connected if $\emptyset_{\boldsymbol{E}}$ and $\boldsymbol{X_E}$ are the only soft ii-open and soft ii-closed sets in the same time in (X,τ,E).

Definition1.8. Let$(\boldsymbol{S},\boldsymbol{E}) \tilde{\in} \boldsymbol{SS}(\boldsymbol{X_E})$, the collection of soft subsets of *X {(J_λ,E)}* is called soft cover for $(\boldsymbol{S},\boldsymbol{E})$ if $(\boldsymbol{S},\boldsymbol{E}) \tilde{\subseteq} \cup_{\boldsymbol{\lambda}} (\boldsymbol{J_\lambda},\boldsymbol{E})$, if the all elements of this cover are soft ii-open sets, Afterward, it is known as a soft ii-open cover. If any part of this cover is a cover for as well *(S,E)*, then it is called a sub-cover of it. If these sub-cover is finite such that,$(\boldsymbol{S},\boldsymbol{E}) \tilde{\subseteq} \cup_{\boldsymbol{i=1}}^{\boldsymbol{n}} (\boldsymbol{J_i},\boldsymbol{E})$, then it is called a finite sub-cover for $(\boldsymbol{S},\boldsymbol{E})$. If from each soft ii-open cover for$(\boldsymbol{S},\boldsymbol{E})$, there exist a finite sub-cover for$(\boldsymbol{S},\boldsymbol{E})$, then $(\boldsymbol{S},\boldsymbol{E})$ is called soft ii-compact set. $(X,\tau,E)$is supposed to be soft ii-compact space if *X* is soft ii-compact.

Definition1.9.[13] Assume that $(X,\tau,E)$is *sTs* and that$(S,E) \tilde{\in} SS(X_E)$. Definition of $\tau_{(\boldsymbol{S},\boldsymbol{E})}$ as$\{(G,E) \tilde{\cap} (S,E) : (G,E) \tilde{\in} \tau$ which took into consideration soft topology on *(S,E)*. The soft topology is referred to as the soft relative topology, and $((S,E), \tau_{(\boldsymbol{S},\boldsymbol{E})})$ is referred to as the soft subspace (soft relative topology of τ on(S,E)) (soft partial subspace) of(X,τ,E).

Theorem1.10. If and only if it is soft ii-connected in a soft partial sub-space ($X^*,\boldsymbol{\tau}*, E$), a soft set *(S,E)* is said to be soft ii-connected in (X,τ,E).

Proof: Assume that $(\boldsymbol{S},\boldsymbol{E}) \tilde{\subseteq} \boldsymbol{X}^* \tilde{\subseteq} \boldsymbol{X}$. Now if ($S_1$,*E*) and ($S_2$,*E*) are non-empty deferent soft sets with $(\boldsymbol{S_1},\boldsymbol{E}) \tilde{\cup} (\boldsymbol{S_2},\boldsymbol{E}) = (\boldsymbol{S},\boldsymbol{E})$. We get, $(\boldsymbol{S1},\boldsymbol{E}),(\boldsymbol{S2},\boldsymbol{E}) \tilde{\subseteq} (\boldsymbol{S},\boldsymbol{E}) \tilde{\subseteq} \boldsymbol{X}^* \tilde{\subseteq} \boldsymbol{X}$.

$(\boldsymbol{sII}-\boldsymbol{Cl}(\boldsymbol{S_1},\boldsymbol{E})\tilde{\cap}(\boldsymbol{S_2},\boldsymbol{E})) \tilde{\cup} ((\boldsymbol{S_1},\boldsymbol{E})\tilde{\cap}\boldsymbol{sII}-\boldsymbol{Cl}(\boldsymbol{S_2},\boldsymbol{E}))$

$=(\boldsymbol{sII}-\boldsymbol{Cl}(\boldsymbol{S_1},\boldsymbol{E})\tilde{\cap}\boldsymbol{X}^*\tilde{\cap}(\boldsymbol{S_2},\boldsymbol{E})) \tilde{\cup} ((\boldsymbol{S_1},\boldsymbol{E})\tilde{\cap}\boldsymbol{X}^*\tilde{\cap}\boldsymbol{sII}-\boldsymbol{Cl}(\boldsymbol{S_2},\boldsymbol{E}))$

$=(\boldsymbol{sII}-\boldsymbol{Cl}^*(\boldsymbol{S_1},\boldsymbol{E})\tilde{\cap}(\boldsymbol{S_2},\boldsymbol{E})) \tilde{\cup} ((\boldsymbol{S_1},\boldsymbol{E})\tilde{\cap}\boldsymbol{sII}-\boldsymbol{Cl}^*(\boldsymbol{S_2},\boldsymbol{E}))$.

Now if $(\boldsymbol{S},\boldsymbol{E})$ is soft ii-connected in ($X,\boldsymbol{\tau}, E$), then $(\boldsymbol{sII}-\boldsymbol{Cl}(\boldsymbol{S_1},\boldsymbol{E})\tilde{\cap}(\boldsymbol{S_2},\boldsymbol{E})) \tilde{\cup} ((\boldsymbol{S_1},\boldsymbol{E})\tilde{\cap}\boldsymbol{sII}-\boldsymbol{Cl}(\boldsymbol{S_2},\boldsymbol{E})) \neq \emptyset$,

Also, $(\boldsymbol{sII}-\boldsymbol{Cl}^*(\boldsymbol{S_1},\boldsymbol{E})\tilde{\cap}(\boldsymbol{S_2},\boldsymbol{E})) \tilde{\cup} ((\boldsymbol{S_1},\boldsymbol{E})\tilde{\cap}\boldsymbol{sII}-\boldsymbol{Cl}^*(\boldsymbol{S_2},\boldsymbol{E})) \neq \emptyset$. Henceforth, $(\boldsymbol{S},\boldsymbol{E})$ is soft ii-connected in ($X^*,\boldsymbol{\tau}*, E$).

Similarly, if $(\boldsymbol{S},\boldsymbol{E})$is soft ii-connected in ($X^*,\boldsymbol{\tau}*,E$), then,$(\boldsymbol{sII}-\boldsymbol{Cl}^*(\boldsymbol{S_1},\boldsymbol{E})\tilde{\cap}(\boldsymbol{S_2},\boldsymbol{E})) \tilde{\cup} ((\boldsymbol{S_1},\boldsymbol{E})\tilde{\cap}\boldsymbol{sII}-\boldsymbol{Cl}^*(\boldsymbol{S_2},\boldsymbol{E})) \neq \emptyset$,

Also, $(\boldsymbol{sII}-\boldsymbol{Cl}(\boldsymbol{S_1},\boldsymbol{E})\tilde{\cap}(\boldsymbol{S_2},\boldsymbol{E})) \tilde{\cup} ((\boldsymbol{S_1},\boldsymbol{E})\tilde{\cap}\boldsymbol{sII}-\boldsymbol{Cl}(\boldsymbol{S_2},\boldsymbol{E})) \neq \emptyset$. Henceforth, $(\boldsymbol{S},\boldsymbol{E})$ is soft ii-connected in (X,τ,E).

Theorem1.11. If (X,τ,E) is a *sTs*, then,$\boldsymbol{X_E} = \langle(\boldsymbol{S_1},\boldsymbol{E})|(\boldsymbol{S_2},\boldsymbol{E})\rangle$ if then just $(\boldsymbol{S_1},\boldsymbol{E})$, $(\boldsymbol{S_2},\boldsymbol{E})$ are deferent sets each of them is *sII-Cs* and *sII-Os*, non-empty and soft subsets of *X*.

Proof: 1. Suppose that$\boldsymbol{X_E} = \langle(\boldsymbol{S_1},\boldsymbol{E})|(\boldsymbol{S_2},\boldsymbol{E})\rangle$, we get, $(\boldsymbol{S_1},\boldsymbol{E})$, $(\boldsymbol{S_2},\boldsymbol{E})$ non-empty soft subsets of *X* and$(\boldsymbol{S_1},\boldsymbol{E})\tilde{\cap}(\boldsymbol{S_2},\boldsymbol{E}) = \emptyset_{\boldsymbol{E}}$, $(\boldsymbol{S_1},\boldsymbol{E}) \tilde{\cup} (\boldsymbol{S_2},\boldsymbol{E}) = \boldsymbol{X_E}$,

Also, $\boldsymbol{sii}-\boldsymbol{Cl}(\boldsymbol{S_1},\boldsymbol{E})\tilde{\cap}(\boldsymbol{S_2},\boldsymbol{E}) = \emptyset_{\boldsymbol{E}}$, $(\boldsymbol{S_1},\boldsymbol{E})\tilde{\cap}\boldsymbol{sii}-\boldsymbol{Cl}(\boldsymbol{S_2},\boldsymbol{E})) = \emptyset_{\boldsymbol{E}}$.

Now, since, $(\boldsymbol{S_1},\boldsymbol{E})\tilde{\cap}\boldsymbol{sii}-\boldsymbol{Cl}(\boldsymbol{S_2},\boldsymbol{E})) = \emptyset_{\boldsymbol{E}}$, we get, $(\boldsymbol{S_2},\boldsymbol{E})$ contains all its ii-limit points, hence $(\boldsymbol{S_2},\boldsymbol{E})$ *is sII-Cs*. Similarly, $(\boldsymbol{S_1},\boldsymbol{E})$ *is sII-Cs*.

Since, $(\boldsymbol{S_2},\boldsymbol{E}) = (\boldsymbol{S_1},\boldsymbol{E})^{\boldsymbol{C}}$, we get, $(\boldsymbol{S_1},\boldsymbol{E})$ *is sII-Os*, Similarly, $(\boldsymbol{S_2},\boldsymbol{E})$ *is sII-Os*.

2. Suppose that$(\boldsymbol{S_1},\boldsymbol{E})$, $(\boldsymbol{S_2},\boldsymbol{E})$ are deferent sets each of them is *sII-Cs* and *sII-Os*, non-empty and soft subsets of *X*, we get, $(\boldsymbol{S_1},\boldsymbol{E}) \neq \emptyset_{\boldsymbol{E}}$, $(\boldsymbol{S_2},\boldsymbol{E}) \neq \emptyset_{\boldsymbol{E}}$, $(\boldsymbol{S_1},\boldsymbol{E})\tilde{\cap}(\boldsymbol{S_2},\boldsymbol{E}) = \emptyset_{\boldsymbol{E}}$.

Since $(\boldsymbol{S_1},\boldsymbol{E})$, $(\boldsymbol{S_2},\boldsymbol{E})$ are *sII-Cs* and *sII-Os*, then, $(\boldsymbol{S_2},\boldsymbol{E}) = (\boldsymbol{S_1},\boldsymbol{E})^{\boldsymbol{C}}$,

$(\boldsymbol{S_1},\boldsymbol{E})^{\boldsymbol{C}} \tilde{\cup} (\boldsymbol{S_1},\boldsymbol{E}) = (\boldsymbol{S_1},\boldsymbol{E}) \tilde{\cup} (\boldsymbol{S_2},\boldsymbol{E})=\boldsymbol{X_E}$.Since, $(\boldsymbol{S_1},\boldsymbol{E})$ is *sII-Cs*, we get,$\boldsymbol{sii}-\boldsymbol{Cl}(\boldsymbol{S_1},\boldsymbol{E}) = (\boldsymbol{S_1},\boldsymbol{E})$, then, $\boldsymbol{sii}-\boldsymbol{Cl}(\boldsymbol{S_1},\boldsymbol{E})\tilde{\cap}(\boldsymbol{S_2},\boldsymbol{E}) = \emptyset_{\boldsymbol{E}}$.

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Similarly, $(S_1, E)\tilde{\cap} sii - Cl(S_2, E)) = \emptyset_E$. Henceforth, $X_E = \langle (S_1, E) | (S_2, E) \rangle$.

Theorem1.12. If$(S, E) \tilde{\subseteq} X^* \tilde{\subseteq} X$, then (S, E) is soft ii-compact in (X,τ,E)if and only if it is soft ii-compact in a soft partial sub-space $(X^*, \tau*, E)$.

Proof: Assume that $(S, E)$is soft ii-compact in a soft partial sub-space $(X^*, \tau*, E)$.

Let $\{(J_\lambda, E)\}$ be a soft ii-open sub-cover for (S, E)in(X,τ,E), that is, $(S, E) \tilde{\subseteq} \cup_\lambda (J_\lambda, E)$.

Since, $(S, E) = (S, E) \tilde{\cap} X^* \tilde{\subseteq} (\cup_\lambda (J_\lambda, E)) \tilde{\cap} X^* = \cup_\lambda ((J_\lambda, E) \tilde{\cap} X^*) = \cup_\lambda (J_\lambda^*, E)$.

We conclude that,$\{(J_\lambda^*, E)\}$ is a soft ii-open sub-cover for(S, E) in$(X^*, \tau*, E)$.

But, (S, E) is soft ii-compact in $(X^*, \tau*, E)$(by suppose). Then there exists a finite soft sub-cover from it, that is$(S, E) \tilde{\subseteq} \cup_{i=1}^{n} (J_i^*, E)$,

$(S, E) \tilde{\subseteq} \cup_{i=1}^{n} ((J_i , E) \tilde{\cap} X^*) \tilde{\subseteq} \cup_{i=1}^{n} ((J_i , E)$. Henceforth, (S, E) is soft ii-compact in(X,τ,E).

Conversely, Assume that $(S, E)$is soft ii-compact in $(X,\tau,E)$and $\{(J_\lambda^*, E)\}$ is a soft ii-open sub-cover for(S, E)in $(X^*, \tau*, E)$, that is, $(S, E) \tilde{\subseteq} \cup_\lambda (J_\lambda^*, E)$.

By "Definition 1.9", we get$(J_\lambda^*, E) = (J_\lambda, E) \tilde{\cap} X^*$for all λ. we have,$\{(J_\lambda, E)\}$ be a soft ii-open sub-cover for (S, E) in(X,τ,E), because

$(S, E) \tilde{\subseteq} \cup_\lambda (J_\lambda^*, E) = \cup_\lambda ((J_\lambda, E) \tilde{\cap} X^*) = (\cup_\lambda (J_\lambda, E)) \tilde{\cap} X^* \tilde{\subseteq} \cup_\lambda (J_\lambda, E)$.

Since (S, E) is soft ii-compact in (X, τ, E), we get $(S, E) \tilde{\subseteq} \cup_{i=1}^{n} ((J_i , E)$.

Now, $(S, E) = (S, E) \tilde{\cap} X^* \tilde{\subseteq} (\cup_{i=1}^{n} ((J_i , E)) \tilde{\cap} X^* = \cup_{i=1}^{n} ((J_i , E) \tilde{\cap} X^*) = \cup_{i=1}^{n} (J_i^*, E)$.

Henceforth, (S, E) is soft ii-compact in$(X^*, \tau*, E)$.

2. Practical Side of Soft Sets and Soft Topologies: An application of Soft Sets

Step 1: We utilize the soft set and soft topology in this application to calculate the cost of infrastructure projects to be completed in numerous residential areas in any city's governorate, as well as to find many data information's that we require.
Let X represents some residential neighbourhoods in the studied city.
$X=\{z1=R1, z2=R2, z3=R3, z4=R4, z5=R5, z6=R6, z7=R7, z8=R8, z9=R9,\dots,z_n=R_n\}$
$W=\{$ $w1$= Project 1, $w2$= Project 2, $w3$= Project 3 , $w4$=Project 4, $w5$= Project 5, $w6$= Project 6 ,….., w_k= Project $k\}$
W represents the infrastructures projects to be accomplished in the governorate of studied city or town.
C_j denotes the cost of the project j, $(j=1, 2, 3, 4, 5, 6,\dots, k)$.

z_{ij} denotes to the number of project *j* in region *i, (i=1,2,3,.....,n), (j=1,2,3,....,k),* $n \neq k$.
L_i denotes the summation of the costs of projects in region *i, i=1,2,3,....,n*.
M_j denotes the costs of project j in all regions *i, i=1,2,3,....,n, j=1,2,3,...,k.*

Cj	C_1	C_2	C_3	…	C_k	L_i
X/W	w_1	w_2	w_3	…	w_k	$L_i=\sum_j z_{ij}.C_j$
z1	z_{11}	z_{12}	z_{13}	…	z_{1k}	L_1
z2	z_{21}	z_{22}	z_{23}	…	z_{2k}	L_2
z3	z_{31}	z_{32}	z_{33}	…	z_{3k}	L_3
……	………	……	……	…		………
z_n	z_{n1}	z_{n2}	z_{n3}	…	z_{nk}	L_n
$M_j = C_j . \sum_{ij} z_{ij}$	M_1	M_2	M_3	…	M_K	$\sum_{i=1}^{n} L_i = \sum_{j=1}^{k} M_j$

This table represents a soft set *(S,W)* that calculates the cost of infrastructure projects in *n* residential zones in the city or town under consideration.

Step 2: Generate the soft topology from the soft sets.

$\tau = \{\emptyset, X, (S_1, W), (S_2, W), (S_3, W), \dots, (S_n, W), (S_{n+1}, W),$
$\{(w_1, \{z_1, z_2\}), (w_2, \{z_1, z_2\}), (w_3, \{z_1, z_2\}), \dots\dots, (w_k, \{z_1, z_2\})\}, \dots,$
$\{(w_1, \{z_1, z_3\}), (w_2, \{z_1, z_3\}), (w_3, \{z_1, z_3\}), \dots, (w_k, \{z_1, z_3\})\}, \dots,$
$\{(w_1, \{z_2, z_3\}), (w_2, \{z_2, z_3\}), (w_3, \{z_2, z_3\}), \dots, (w_k, \{z_2, z_3\})\}, \dots,$
$\{(w_1, \{z_{n-1}, z_n\}), (w_2, \{z_{n-1}, z_n\}), (w_3, \{z_{n-1}, z_n\}), \dots, (w_k, \{z_{n-1}, z_n\})\}\}.$

Where, $(S_1, W)=\{(w_1, \{z_1\}), (w_2, \{z_1\}), (w_3, \{z_1\}), \dots, (w_k, \{z_1\})\}$

(S_1, W) Computes the cost of infrastructure projects to be accomplished in the first residential area z1 in the studied city, which equal to:

$(z_{11} * C_1) + (z_{12} * C_2) + (z_{13} * C_3) + \cdots + (z_{1k} * C_k) = L_1$……………. (1)

$(S_2, W)=\{(w_1, \{z_2\}), (w_2, \{z_2\}), (w_3, \{z_2\}), \dots, (w_k, \{z_2\})\}$

(S_2, W) Compute the cost of infrastructure projects to be accomplished in the second residential area z2 in the studied city, which equal to:

$(z_{21} * C_1) + (z_{22} * C_2) + (z_{23} * C_3) + \cdots + (z_{2k} * C_k) = L_2$…………… (2)

$(S_3, W)=\{(w_1, \{z_3\}), (w_2, \{z_3\}), (w_3, \{z_3\}), \dots, (w_k, \{z_3\})\}$

(S_3, W) Computes the cost of infrastructure projects to be accomplished in the third residential area z3 in the studied city, which equal to:

$(z_{31} * C_1) + (z_{32} * C_2) + (z_{33} * C_3) + \cdots + (z_{3k} * C_k) = L_3$……………..... (3)

. .
. .
. .

$(S_n, W)=\{(w_1, \{z_n\}), (w_2, \{z_n\}), (w_3, \{z_n\}), \dots, (w_k, \{z_n\})\}$

(S_n, W) Computes the cost of infrastructure projects to be accomplished in the residential area z_n in the studied city, which equal to:

$(z_{n1} * C_1) + (z_{n2} * C_2) + (z_{n3} * C_3) + \cdots + (z_{nk} * C_k) = L_n$……………..... (n)

The cost of infrastructure projects to be accomplished in n areas (z_1, z_2, z_3,…..,z_n) in the studied city, which equal to:

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$$L_1 + L_2 + L_3 + \cdots + L_n = \sum_{i=1}^{n} L_i$$

Let (S_{n+1}, W)={$(w_1, \{z_1, z_2, z_3, \dots, z_n\}), (w_2, \{z_1, z_2, z_3, \dots, z_n\}), (w_3, \{z_1, z_2, z_3, \dots, z_n\}),$
$, \dots, (w_k, \{z_1, z_2, z_3, \dots, z_n\})\}$

(S_{n+1}, W) Computes the cost of infrastructure projects to be accomplished in the n residential areas $(z_1, z_2, z_3, \dots, z_n)$ in the studied city, which equal to:

$$C_1.(z_{11} + z_{21} + z_{31} + \cdots + z_{n1}) + C_2.(z_{12} + z_{22} + z_{32} + \cdots + z_{n2})$$
$+C_3.(z_{13} + z_{23} + z_{33} + \cdots + z_{n3}) + \cdots + C_k.(z_{1k} + z_{2k} + z_{3k} + \cdots + z_{nk})$……. (n+1)
$=M_1 + M_2 + M_3 + \cdots + M_k = \sum_{j=1}^{k} M_j$ which equal to $\sum_{i=1}^{n} L_i$.

Now we will give the following example of "Application 1"

Example1.1

Step1: We utilize the soft set and soft topology in this example to compute the cost of infrastructure projects to be completed in several residential areas in the governorate of Telkaif town, as well as to identify numerous data information's that we require.

Let *X* represents some residential neighborhoods in Telkaif town.

X={z1= Miskalat, *z2*= Baqofa, *z3*= Jargan, *z4*= Telleskuf, *z5*= Batnaya}

W={ *w1*= Replacing and Extending Water Networks, *w2*= Building a Well Room, *w3*= Building an Iron Water Tank, *w4*= Building a Rain Water Stream}

W represents the infrastructures projects to be accomplished in the governorate of Telkaif Town. *j={1,2,3,4}, i={1,2,3,4,5}*

Cj	*121000000*	*6000000*	*16000000*	*200000000*	L_i
X/W	*w1*	*w2*	*w3*	*w4*	$L_i=(\sum_j z_{ij}).C_j$
z1	*2*	*0*	*1*	*0*	L_1=258000000
z2	*1*	*0*	*0*	*0*	L_2=121000000
z3	*1*	*1*	*1*	*0*	L_3=143000000
z4	*1*	*3*	*1*	*1*	L_4=355000000
z5	*1*	*2*	*1*	*1*	L_5=349000000
$M_j = C_j.(\sum_{ij} z_{ij})$	M_1= 726000000	M_2=36000000	M_3=64000000	M_4=400000000	$L_1+L_2+L_3+L_4+L_5$ =1226000000 $=M_1+M_2+M_3+M_4$

This table represents a soft set(S, W) which compute the cost of infrastructure projects to be accomplished in five residential areas in the governorate of Telkaif town.

Step 2: Generate the soft topology from the soft sets.

$\tau = \{\emptyset, X, (S_1, W), (S_2, W), (S_3, W), (S_4, W), (S_5, W), (S_6, W),$
$\{(w_1, \{z_1, z_2\}), (w_2, \{z_1, z_2\}), (w_3, \{z_1, z_2\}), (w_4, \{z_1, z_2\})\}$
$,\{(w_1, \{z_1, z_3\}), (w_2, \{z_1, z_3\}), (w_3, \{z_1, z_3\}), (w_4, \{z_1, z_3\})\}$,……….
…….,$\{(w_1, \{z_3, z_5\}), (w_2, \{z_3, z_5\}), (w_3, \{z_3, z_5\}), (w_4, \{z_3, z_5\})\}$,
$\{(w_1, \{z_4, z_5\}), (w_2, \{z_4, z_5\}), (w_3, \{z_4, z_5\}), (w_4, \{z_4, z_5\})\}\}$.

Where, (S_1, W)={$(w_1, \{z_1\}), (w_2, \{z_1\}), (w_3, \{z_1\}), (w_4, \{z_1\})\}$, (S_1, W) computes the cost of infrastructure projects to be accomplished in the first residential area *z1* in the governorate of Telkaif town, which equal to:

$L_1 = (2 * 121000000) + (0 * 6000000) + (1 * 16000000) + (0 * 200000000) = 258000000$.
(S_2, W)={$(w_1, \{z_2\}), (w_2, \{z_2\}), (w_3, \{z_2\}), (w_4, \{z_2\})$} , (S_2, W) computes the cost of infrastructure projects to be accomplished in the second residential area *z2* in the governorate of Telkaif city, which equal to:
$L_2 = (1 * 121000000) + (0 * 6000000) + (0 * 16000000) + (0 * 200000000) = 121000000$.
(S_3, W)={$(w_1, \{z_3\}), (w_2, \{z_3\}), (w_3, \{z_3\}), (w_4, \{z_3\})$}, (S_3, W) computes the cost of infrastructure projects to be accomplished in the third residential area *z3* in the governorate of Telkaif town, which equal to:
$L_3 = (1 * 121000000) + (1 * 6000000) + (1 * 16000000) + (0 * 200000000) = 143000000$.
(S_4, W)={$(w_1, \{z_4\}), (w_2, \{z_4\}), (w_3, \{z_4\}), (w_4, \{z_4\})$}, (S_4, W) computes the cost of infrastructure projects to be accomplished in the fourth residential area *z4* in the governorate of Telkaif town, which equal to:
$L_4 = (1 * 121000000) + (3 * 6000000) + (1 * 16000000) + (1 * 200000000) = 355000000$.
(S_5, W)={$(w_1, \{z_5\}), (w_2, \{z_5\}), (w_3, \{z_5\}), (w_4, \{z_5\})$} , (S_5, W) computes the cost of infrastructure projects to be accomplished in the fifth residential area *z5* in the governorate of Telkaif town, which equal to:
$L_5 = (1 * 121000000) + (2 * 6000000) + (1 * 16000000) + (1 * 200000000) = 349000000$.
The cost of infrastructure projects to be accomplished in the five areas *(z1, z2, z3, z4 and z5)* in the governorate of Telkaif town, which equal to:
$L_1 + L_2 + L_3 + L_4 + L_5 = 258000000 + 121000000 + 143000000 + 355000000 + 349000000 = 1226000000.$
Let (S_6, W)={$(w_1, \{z_1, z_2, z_3, z_4, z_5\}), (w_2, \{z_1, z_2, z_3, z_4, z_5\}), (w_3, \{z_1, z_2, z_3, z_4, z_5\}), (w_4, \{z_1, z_2, z_3, z_4, z_5\})$}.
(S_6, W) Computes the cost of infrastructure projects to be accomplished in the five residential areas (z1, z2, z3, z4 and z5) in the governorate of Telkaif town, which equal to:

$$C_1.(z_{11} + z_{21} + z_{31} + z_{41} + z_{51}) + C_2.(z_{12} + z_{22} + z_{32} + z_{42} + z_{52}) + C_3.(z_{13} + z_{23} + z_{33} + z_{43} + z_{53}) + C_k.(z_{14} + z_{24} + z_{34} + z_{44} + z_{54})$$

$= (((2 + 1 + 1 + 1 + 1) * 121000000)) + ((0 + 0 + 1 + 3 + 2) * 6000000)) + \big((1 + 0 + 1 + 1 + 1) * 16000000\big)) + ((0 + 0 + 0 + 1 + 1) * 200000000)))$= $M_1+M_2+M_3+M_4$= 1226000000 .
Which equal to the sum of the costs which compute by$(S_1, W), (S_2, W), (S_3, W), (S_4, W) and\ (S_5, W)$ which equal to$L_1 + L_2 + L_3 + L_4 + L_5$.

3. Conclusions:

From the example mentioned above, we conclude that we can use the theory of soft sets which we explained a part of it in the theoretical part in many fields in practical and real life, which makes work easier, fun, productive and efficient.

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