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The Effect of a Magnetic Field on the Stability of Fluid Flow in a Porous Channel

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Abstract. In this article, we investigate the stability of flowing fluid in a horizontal cavity. The walls of which consists of a porous material under the influence of a magnetic field (MF) perpendicular to the cavity plane. We find that the real part of the wave velocity ($\boldsymbol{\alpha}$) is negative for any value of the Hartmann number ($\boldsymbol{Ha}$) and the Ghratshof number ($\boldsymbol{Gr}$) is greater than zero. In addition, we convert the governing equations to non-dimensional formulas. So, we obtain that Reynolds number ($\boldsymbol{Re}$), wave number (k), Hartmann number ($\boldsymbol{Ha}$), Ghratshof number ($\boldsymbol{Gr}$), and Darcy number ($\boldsymbol{Da}$) affect on the stability of the model through the graphs of these numbers
Keywords. Stability Analysis, Magneic field, Wave number, Hartmann number.

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1. Introduction

Fluid mechanics is one of the engineering disciplines that constitute the foundations of the other engineering generals as whole Branches into various disciplines such as aerodynamics, hydraulic engineering, ship engineering and others. Since the fluid moves under the influence of an unbalanced force affecting it, the nature of the flow of a real fluid is very complex Since the basic laws that describe the quantum motion of a fluid are not formulated in a proper way, and mathematical loads need to be the use of practical experiences. The study of the movement of the fluid leads us to the study of the stability of this fluid. If a running fluid is disturbed, it may disappear this disturbance with the passage of time fades away and the properties of the fluid return to their previous condition, and this disturbance may increase with the passage of time turns into a turbulent fluid, and this turbulence may increase and lead to a complete change in the properties of the fluid, which is what It's called anarchy. When the slop angle of the channel is 30 °, Hammodat A. and Jumaa A. They discussed the issue of fluid dynamics in a permeable medium under the impact of a MF orthogonal to the channel plane in 2013. It should be noted that changes in the values of the Brickman number Fs, wave number k, and pouger number w have an effect on the system's stability [1]. During 2014, Hammodat A., and Hamdon T., discuss stability analysis in horizontal and inclined glass cavities and when the amplitudes are variable by perturbing the governing equations. This evaluation is carried out by determining the system's eigenvalues, which allows us to determine whether or not the perturbation is growing. They deduce that the model are stable when the real part of the wave velocity is negative and unstable when it is positive [2].In the same year, the analytical solution to the problem of three dimensional stable magneto-hydrodynamic flow (MHD) of a fluid flowing in a vertical channel whose walls are made of porous material and open on one side was found by Jabr and Abdulhadi [3In 2016 [4], Mahantesh M. Shilpa J. discussed numerical calculations for the friction factor, locally Nusselt for the matter of flow at the point of stationary for a nanofluid flow, and heat flux on a stretching/shrinking plate in a quasi media. In 2017, Kumari K. and Goyal M. investigated the effects of radiant heat transfer and transverse magnetization on the erratic stream of a visually conductive fluid in a porous layer with uneven walls. According to the findings, increasing the magnetic field strength decreased the wall shear stress while increasing the radiation parameter increased it [5]. Al-Amin et al. 2018 [6] offered a simulation of the issue of the magnetic field's effect on fluids and heat conduction in fixed porosity cavities while accounting for the variable in nanofluid concentration. Rahman A. and et al looked into the magnetohydrodynamic (MHD) air flow in a rectangular channel in 2019. They arrived at a theoretical resolution to the flow's instability using the energy slope theory. They discussed the mechanism that determines whether the flow in a channel is stable [7]. In 2020, Hammodat A. and Alobaidy I. investigated the stability of the magnetic flux and thermal radiation, as well as the flow and heat transfer systems in horizontal channels when the capacities are changing. In order to solve the computational problem of fluid transferring heat via diffusion and radiation in a rectangular layer while being subjected to an electromagnetic field (EMF), Hammodat A. and et al. applied the alternative directed implicit technique (ADI) by the year 2021 [9]. Adeshina T. and Joel C. describe in 2022 [10] how different electrical properties affect the ability of fluids in an exergonic, responsive hydrodynamic flow to adsorb substances across layers. In this paper, we provide the steadiness analysis of a class of partial differential equations in a horizontal channel that has been perturbed. This analysis was carried out by determining the eigenvalues of the system that allow us to determine whether or not there is growth turbulence after making the equations linear, and the results show that these equations are steady when the real portion of the wave rapidity is a adverse and uneven when optimistic. In addition, we found the effect of the physical quantities that appeared in the problem graphically using Maple program.

2. Mathematical Model and Problem Formulation:

Figure 1, shows the geometry of the problem studied in the Cartesian coordinate system. Consider the unstable two-dimensional fluid flowing laminarly running in a porous station the planetary among its parapets is $\boldsymbol{y} = \boldsymbol{h}$. The bottom walls temperature is $\boldsymbol{T} = \boldsymbol{T_0}$, and the concentration level $\boldsymbol{C} = \boldsymbol{C_0}$, whereas the upper walls temperature is $\boldsymbol{T} = \boldsymbol{T_1}$, and the concentration level $= \boldsymbol{C_1}$, with $\boldsymbol{T_1}, \boldsymbol{C_1}$ being higher than$\boldsymbol{T_0}$, $\boldsymbol{C_0}$ respectively. The $\boldsymbol{x}$-axis runs parallel to the wall in the flow direction, while the $\boldsymbol{y}$-axis runs perpendicular to it, while the $\mathbf{z}-\mathbf{axis}$ is in the track that the extra dual axes are commonly orthogonal to it. In the model being considered, the rapidity $\boldsymbol{u}$ and $\boldsymbol{v}$ are zero at the advantage, in addition to the magnetic flux has a factor $\mathbf{B_x}$ tempted to move in the flow's track along on the network, $\boldsymbol{B_z}$ is zero, and the component parallel to the y-axis is meant by $\mathbf{B_0}$.The equations that regulate the problem can be written as follows under these assumptions:

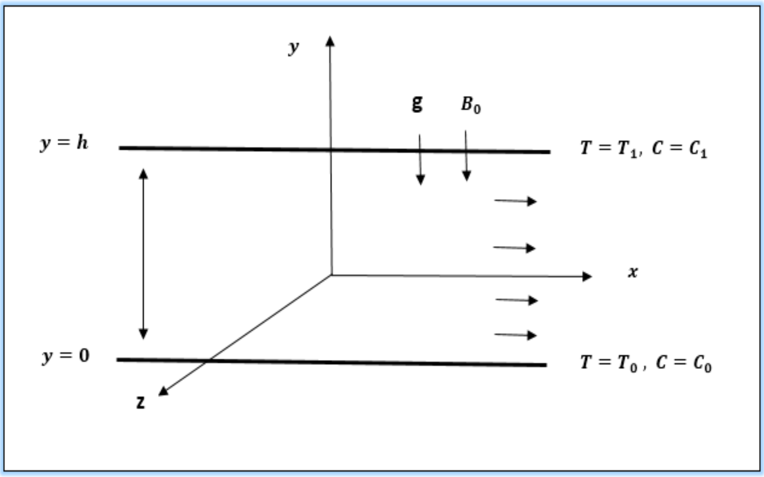

Figure 1. Physical configuration and coordinate system.

Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

Naiver- Stocks equation

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{-1}{\rho}\frac{\partial p}{\partial x}+v\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\frac{v}{K}u \quad (2)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}==\frac{-1}{\rho}\frac{\partial p}{\partial y}+v\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{1}{\rho}\sigma B_0^2 v-g\beta(T-T_1)-g\beta^*(C-C_1) \quad (3)$$

Energy Equation

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k^*}{\rho C_p}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\frac{\mu}{\rho C_p}\left[\left(\frac{\partial u}{\partial y}\right)^2\right] \quad (4)$$

Concentration equation

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right) \quad (5)$$

With the corresponding initial and boundary situations are:

$$\left.\begin{array}{l} u=v=0 \\ T=T_0,T_1\ ,\dfrac{\partial T}{\partial y}=0 \qquad \text{at} \qquad y=0,h \\ C=C_0,C_1\ ,\dfrac{\partial C}{\partial y}=0 \end{array}\right\} \quad (6)$$

Where $u,v,\rho,p,v,K,\sigma,g,\beta,\beta^*,k^*,C_p,\mu,D$ are velocity components in the x,y directions, density, pressure, kinematic viscosity, permeability of porous medium, electrical conductivity, gravity, thermal expansion coefficient, coefficient of volumetric expansion, thermal efficiency, particular heat at constant pressure, viscosity, Darcy factor correspondingly. According to the theory of lubrication, the term $\frac{\partial^2 u}{\partial x^2}$ is negligible compared to the term $\frac{\partial^2 u}{\partial y^2}$ because of $\frac{\partial^2 u}{\partial x^2} \ll \frac{\partial^2 u}{\partial y^2}$ [10]. Therefore, the equations (2), (3), (4), and (5) can be simplified as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (7)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{-1}{\rho}\frac{\partial p}{\partial x}+v\left[\frac{\partial^2 u}{\partial y^2}\right]-\frac{v}{K}u \quad (8)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=\frac{-1}{\rho}\frac{\partial p}{\partial y}+v\left[\frac{\partial^2 v}{\partial y^2}\right]-\frac{1}{\rho}\sigma B_0^2 v-g\beta(T-T_1)-g\beta^*(C-C_1) \quad (9)$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k^*}{\rho C_p}\left[\frac{\partial^2 T}{\partial y^2}\right]+\frac{\mu}{\rho C_p}\left[\left(\frac{\partial u}{\partial y}\right)^2\right] \quad (10)$$

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\left[\frac{\partial^2 C}{\partial y^2}\right] \quad (11)$$

3. Dimensional Analysis

Making the governing equations (7)-(11) dimensionless is essential to determine the stability of their solutions under the beginning and boundary conditions (6). Introduce the dimensionless quantities shown below for this purpose [11].

$$\left.\begin{array}{llll} U=\frac{uh}{v\sqrt{Gr}}\ , & V=\frac{vh}{v\sqrt{Gr}}\ , & \theta=\frac{T-T_1}{T_1-T_0}\ , & \phi=\frac{C-C_1}{C_1-C_0} \\ \tau=\frac{tv\sqrt{Gr}}{h^2}\ , & X=\frac{x}{h}\ , & Y=\frac{y}{h}\ , & p=P\rho u^2 \end{array}\right\} \quad (12)$$

After substituting (12) in equations (7), (8), (9), (10) and (11), we get

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{13}$$

$$\frac{\partial U}{\partial \tau}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=\frac{-(Re)^2}{Gr}\frac{\partial P}{\partial X}+\frac{1}{\sqrt{Gr}}\left[\frac{\partial^2 U}{\partial Y^2}\right]-\frac{1}{Da\sqrt{Gr}}U \tag{14}$$

$$\frac{\partial V}{\partial \tau}+U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=\frac{-(Re)^2}{Gr}\frac{\partial P}{\partial Y}+\frac{1}{\sqrt{Gr}}\left[\frac{\partial^2 V}{\partial Y^2}\right]-\left(\frac{(Ha)^2}{\sqrt{Gr}}\right)V-\theta-\left(\frac{Gr^*}{Gr}\right)\phi \tag{15}$$

$$\frac{\partial \theta}{\partial \tau}+U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\left(\frac{1}{Pr\sqrt{Gr}}\right)\frac{\partial^2 \theta}{\partial Y^2}+\varepsilon N\sqrt{Gr}\left[\left(\frac{\partial U}{\partial Y}\right)^2\right] \tag{16}$$

$$\frac{\partial \phi}{\partial \tau}+U\frac{\partial \phi}{\partial X}+V\frac{\partial \phi}{\partial Y}=\frac{1}{Sc\sqrt{Gr}}\left[\frac{\partial^2 \phi}{\partial Y^2}\right] \tag{17}$$

Where $Re, Gr, Da, Ha, Gr^*, Pr, \varepsilon, N, Sc$ are Reynold number, Gratshof number, Darcy number, Hartmann number, Grattshof number for mass transfer, Prandtl number, dispersion parameter, new physical quantity, and Schmidte number respectively.

4. Stability Analysis

To study the problem in greater depth and to provide a clear picture of the model, the functions of velocities, pressure, temperature, and diffusion must take the following convenient form [11]:

$$\left.\begin{aligned} U &= U_1(x,y)+U_2(x,y,t)\\ V &= V_1(x,y)+V_2(x,y,t)\\ \theta &= \theta_1(x,y)+\theta_2(x,y,t)\\ \phi &= \phi_1(x,y)+\phi_2(x,y,t)\\ P &= P_1(x,y)+P_2(x,y,t)\end{aligned}\right\} \tag{18}$$

Where $U_1, V_1, \theta_1, \phi_1, P_1$ are independent of t.
To get the steady and unsteady equations we will substitute equation (18) into equations (13)-(17). Assuming that the nonlinear part is small compared to the linear part, so we neglected it. Then the linear unstable equations will be in the following form:

$$\frac{\partial U_2}{\partial X}+\frac{\partial V_2}{\partial Y}=0 \tag{19}$$

$$\frac{\partial U_2}{\partial \tau}=\frac{-(Re)^2}{Gr}\frac{\partial P_2}{\partial X}+\frac{1}{\sqrt{Gr}}\left[\frac{\partial^2 U_2}{\partial Y^2}\right]-\frac{1}{Da\sqrt{Gr}}U_2 \tag{20}$$

$$\frac{\partial V_2}{\partial \tau}=\frac{-(Re)^2}{Gr}\frac{\partial P_2}{\partial Y}+\frac{1}{\sqrt{Gr}}\left[\frac{\partial^2 V_2}{\partial Y^2}\right]-\left(\frac{(Ha)^2}{\sqrt{Gr}}\right)V_2-\theta_2-\left(\frac{Gr^*}{Gr}\right)\phi_2 \tag{21}$$

$$\frac{\partial \theta_2}{\partial \tau}=\left(\frac{1}{Pr\sqrt{Gr}}\right)\frac{\partial^2 \theta_2}{\partial X^2} \tag{22}$$

$$\frac{\partial \phi_2}{\partial \tau}=\left(\frac{1}{Sc\sqrt{Gr}}\right)\left[\frac{\partial^2 \phi_2}{\partial Y^2}\right] \tag{23}$$

With the boundary conditions:

$$\left.\begin{aligned} &U_2=V_2=0\\ &\theta_2=0,\ \frac{\partial \theta_2}{\partial y}=0 \qquad at \qquad y=0,1\\ &\phi_2=0,\ \frac{\partial \phi_2}{\partial y}=0\end{aligned}\right\} \tag{24}$$

5. Analysis of Fixed Variance Equilibrium

The effort to discovery the resolution of the equations (19-24),we linearized system and since the coefficient in the various equations is independent [2].

$$\left.\begin{aligned} U_2 &= A\,e^{-ikx}\,e^{\alpha t} \\ V_2 &= B\,e^{-ikx}\,e^{\alpha t} \\ P_2 &= D\,e^{-ikx}\,e^{\alpha t} \\ \theta_2 &= E\,e^{-ikx}\,e^{\alpha t} \\ \phi_2 &= F\,e^{-ikx}\,e^{\alpha t} \end{aligned}\right\} \qquad (25)$$

Where $A, B, D, E, and\ F$ is constant amplitude functions [6], k is wave number, and α is complex number which has the form $= \alpha_1 + \alpha_2$, $\alpha_1, \alpha_2 \in R$ is speed number. From equations (19)-(24), and (25), and since $e^{-ikx}\, e^{\alpha t} \neq 0$, we get:

$$(-ik)A = 0 \qquad (26)$$

$$\left(\alpha + \frac{1}{Da\sqrt{Gr}}\right)A - \left(\frac{ik(Re)^2}{Gr}\right)D = 0 \qquad (27)$$

$$\left(\alpha + \frac{(Ha)^2}{\sqrt{Gr}}\right)B + E + \left(\frac{Gr^*}{Gr}\right)F = 0 \qquad (28)$$

$$(\alpha)E = 0 \qquad (29)$$

$$(\alpha)F = 0 \qquad (30)$$

Therefore, these equations (26)-(30) can be solved by the determinant of the following matrix$|X| = 0$. We found that the wave speed$\alpha = -\frac{(Ha)^2}{\sqrt{Gr}}$, like a corollary, the systems is steady for all Hartmann quantities and that when the Gratshof number is higher than zero. In addition, it is possible to find the eigenvalues of the matrix of equations (26)-(30) by finding the determinant of the matrix$|X - \lambda I| = 0$. Including:

$$f(\lambda) = \lambda^5 - (2\alpha - ik)\lambda^4 - (2\alpha ik - \alpha^2 + JN)\lambda^3 - \ldots$$
$$\ldots\_(kiJN - \alpha^2 ik - 2\alpha JN)\lambda^2 - (\alpha^2 JN)\lambda - ki\alpha JN = 0 \qquad (31)$$

Where $N = \left(-\frac{Re^2 ik}{Gr}\right)$, $J = \left(\alpha + \frac{(Ha)^2}{\sqrt{Gr}}\right)$

Now, using (Maple 11) [12], we solve Equation (31) numerically to discover the origins of these calculations, as exposed in data (2-7).

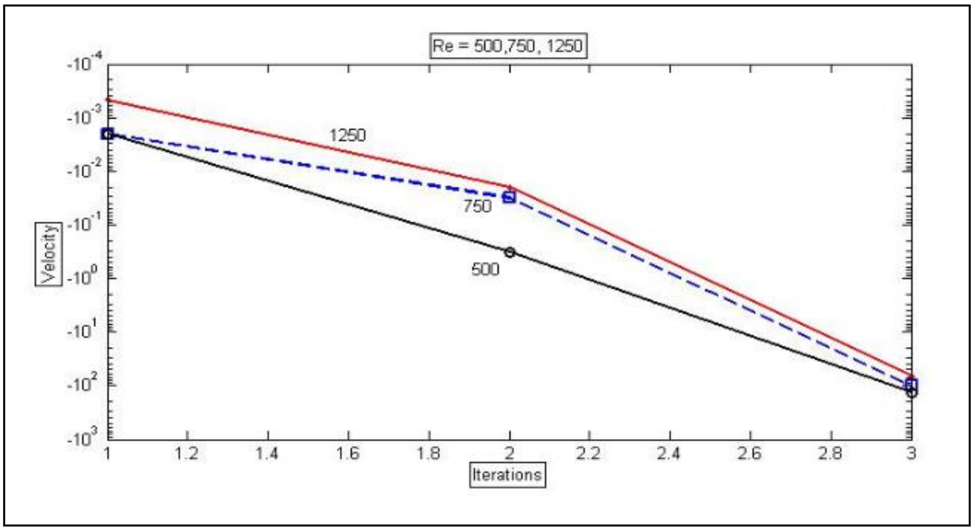

Figure 2. Reynolds number affect (Re = 500, 750, 1250).

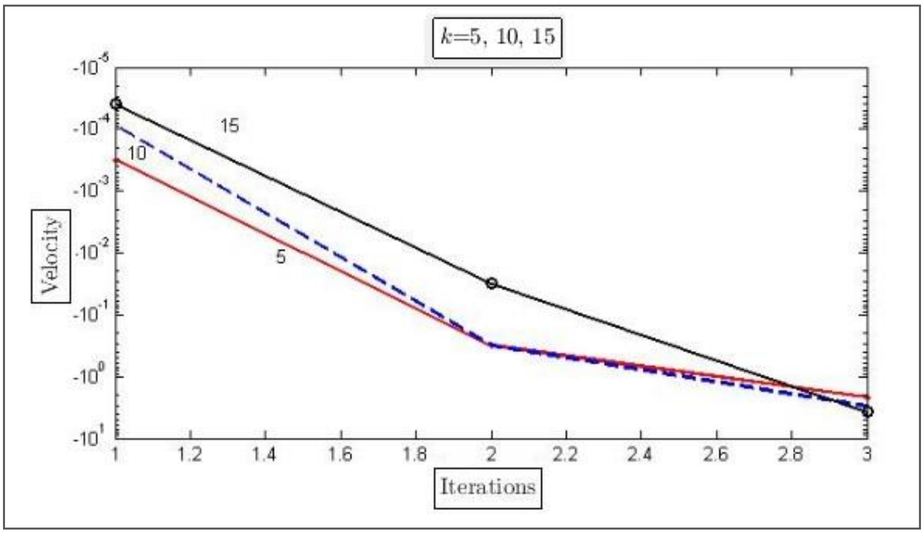

Figure 3. Consequence of Wave number (k = 5, 10, 15).

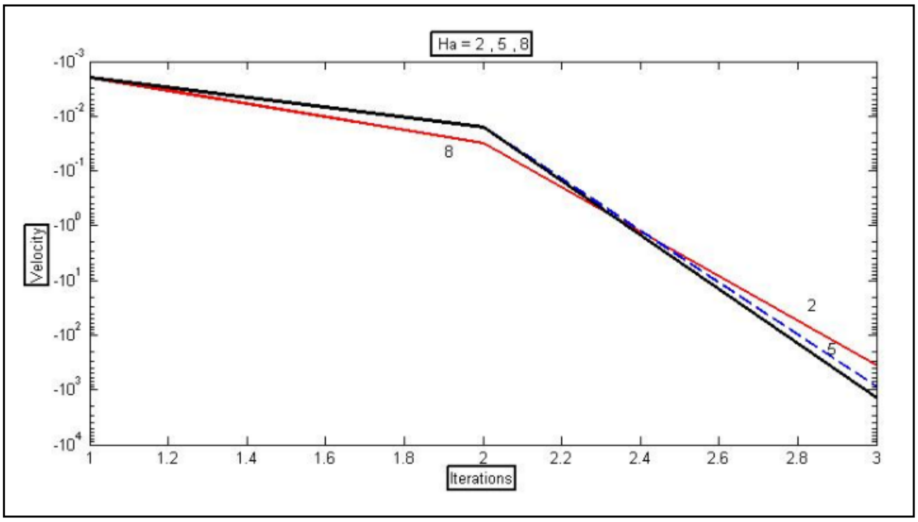

Figure 4. The Hartmann number's impact (Ha= 2, 5, 8).

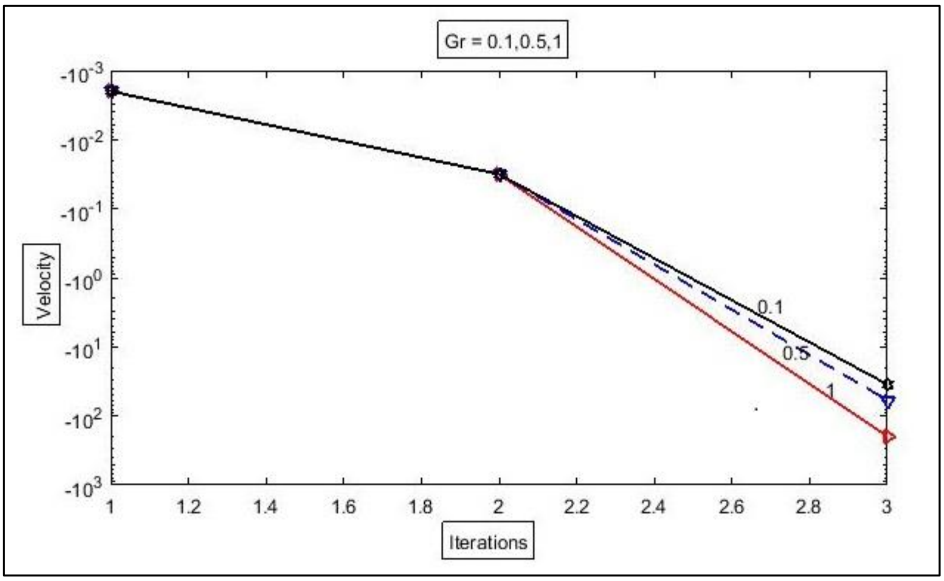

Figure 5. Grashof number's significance for heat flux (Gr = 0.1, 0.5, 1).

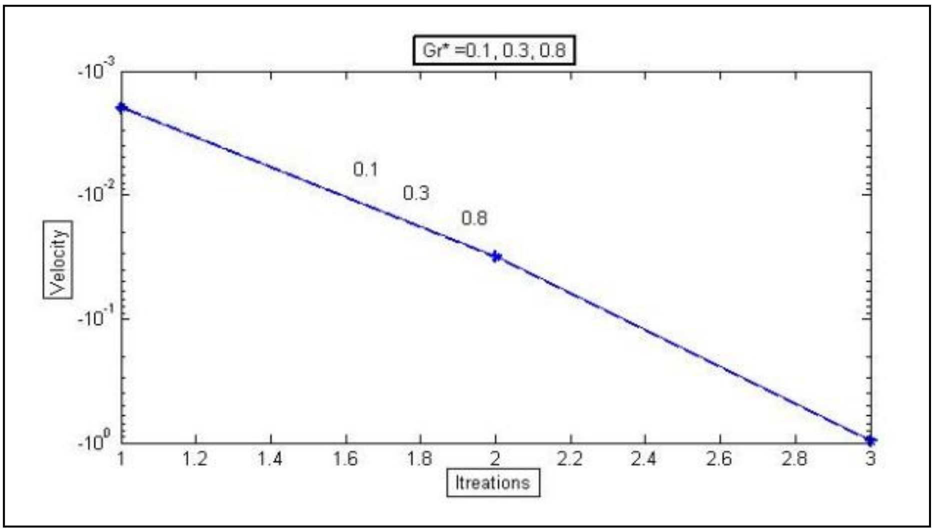

Figure 6. Grashof number's influence on mass transfer (Gr* = 0.1, 0.3, 0.8).

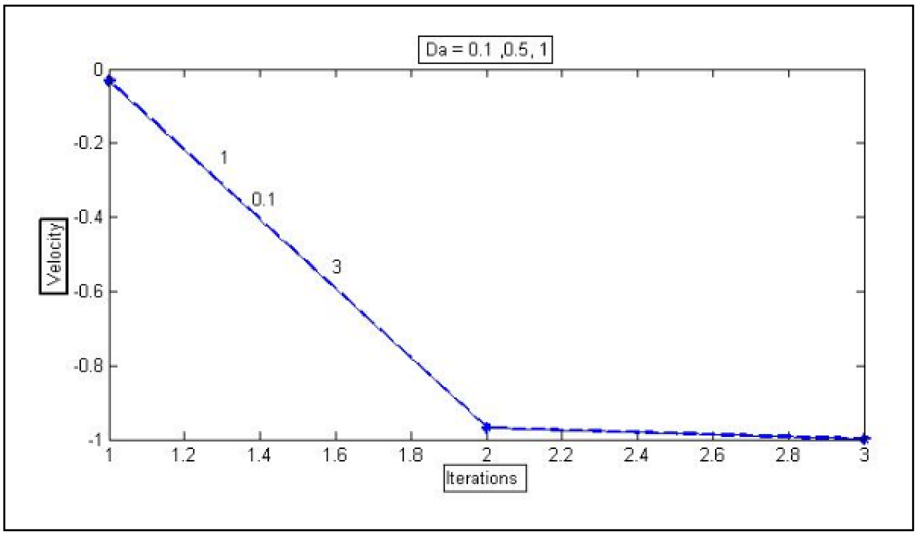

Figure 7. Effects of the Darcy number (Da = 0.1, 0.5, 1).

6. Results and Discussion

The fluid flow stability between two parallel plates and under affect of magnetic field (MF) was studied in this work. was studied in this work. The stability analysis was carried out using Fourier method and when the amplitudes are constant. We noticed from the analytical results that the system is stable at any Hartmann number's ($\mathbf{Ha}$) value and when Gratshof number ($\mathbf{Gr}$) is more than zero. As for the numerical results, we used the Maple (11) program, to find the affect of physical quantities on the stability of the problem, as it is shown in Figures (2-7).

7. Conclusions

The above research begins by introducing design of fluid flow of energy transfer by convection and conduction, and often a system of partial differential equations specifying flow behavior around sequential horizontal wall surfaces separated by a good distance, heat source under it and affected by a magnetic field tangent to the medium. Because the real part of the velocity number α is negative for any value of the Hartmann number and when the Gratshof number is positive, it plays a big part in the stability of the equations of motion, energy, and diffusion when evaluating them analytically. Also, the higher the Reynolds number $\boldsymbol{Re}$, the further away from stability, as shown in Figure (2). From Figure (3), the higher the wave number $\boldsymbol{k}$, the faster the stability is reached, while the increase in the Hartmann number $\boldsymbol{Ha}$ leads to the deviation from stability, as shown in Figure (4). From Figure (5), the higher the Gratshof number $\boldsymbol{Gr}$, the more we reach the steady state. Figures (6) and (7) show that the effect of the Gratshof number for mass transfer $\boldsymbol{Gr}^*$, and Darcy's number $\boldsymbol{Da}$ is very slight on the stability of the problem.

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